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# NON-EQUILIBRIUM THERMODYNAMICS FRAMEWORK FOR FLUID FLOW AND POROSITY DYNAMICS IN POROUS ISOTROPIC MEDIA*

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Dedicated to Dr. Vasile Drăgan on the occasion of his 70$^{\text{th}}$ anniversary

**Abstract**

In previous papers in the linear and anisotropic case, constitutive relations, rate equations, temperature and energy equations were derived by the authors to describe the mechanical, thermal and transport properties of fluid-saturated crystals with porous channels defects, using a model developed by one of us (L. R.) in the framework of non-equilibrium thermodynamics. A structural permeability tensor à la Kubik, $r_{ij}$, its gradient and its flux $\mathcal{V}_{ijk}$ were introduced as internal variables in the thermodynamic state vector. Here, we work out in the isotropic and perfect isotropic linear cases the constitutive functions for the stress tensor, the entropy density, the chemical potentials, and also the rate equations for $r_{ij}$, $\mathcal{V}_{ijk}$, the fluid-concentration and the heat fluxes, describing disturbances propagating with finite velocity

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and presenting a relaxation time. The porous defects modify the thermal conductivity and when they have a density higher than a suitable characteristic value the thermal conductivity decreases. Furthermore, the closure of the system of equations, describing the media under consideration and linearized around a thermodynamic equilibrium state is obtained. The derived results may have great relevance in biology, medical sciences and in several technological sectors, like seismic engineering and nanotechnology (where high-frequency waves propagation is present and the properties variation rate of the considered medium is faster than the relaxation times of the fluxes towards their equilibrium value).
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**keywords:** Porous solids, Non-equilibrium thermodynamics with internal variables, Constitutive relations for porous isotropic media, Rate equations for porous isotropic media.

## 1 Introduction

The study of media with porous defects may have relevance in the description of phenomena accompanying flows of mass in porous structures and find applications in applied sciences. Here, we use a thermodynamic theory (see [1], [2] and also [3], [4]), developed in the framework of Extended Irreversible Thermodynamics, [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], with internal variables. More precisely, in [1] and [2] for the media under consideration the basic equations were established, the Liu's theorem [17] was applied and in a special case the constitutive theory and the rate equations for the fluxes and the porosity field were constructed as objective functions using Smith's theorem [18]. In [3] and [4] constitutive relations, rate equations and other resutls were derived for the same media in the anisotropic case. In this paper we investigate the behaviour of isotropic and perfect isotropic porous structures filled by a fluid flow, having a particular spatial symmetry properties, using a mathematical theory for isotropic cartesian tensors [19], [20]. The influence of porous channels on the other fields, occurring inside the considered medium, is described by a stuctural permeability tensor à la Kubik [21], giving a macroscopic characterization of the porous matrix. In [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32] and [33] models, with some applications, for media with defects having the form of a network of very thin tubes, like porous channels and dislocations, were formulated, using the same methods of non-