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CONSTRAINING TRIANGULATION TO LINE SEGMENTS: A FAST METHOD FOR CONSTRUCTING CONSTRAINED DELAUNAY TRIANGULATION*

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Dedicated to Dr. Vasile Drăgan on the occasion of his 70th anniversary

Abstract

In this paper we present an edge swapping approach for incorporating line segments into triangulation. If the initial triangulation is Delaunay, the algorithm tends to produce optimal Constrained Delaunay triangulation by improving the triangles' aspect ratios from the local area being constrained. There are two types of methods for constructing Constrained Delaunay Triangulation: straight-forward ones which take both points and line segments as source data and produce constrained triangulation from them at once; and post-processing ones which take an already constructed triangulation and incorporate line segments into it. While most of the existing post-processing approaches clear the triangle's edges intersected by the line segment being incorporated and fill the opened hole (cavity) by re-triangulating it, the only processing that our algorithm does is to change the triangulation

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connectivity and to improve the triangles' aspect ratios through edge swapping. Hereof, it is less expensive in terms of both operating and memory costs. The motivation behind our approach is that most of the existing straight-forward triangulators are too slow and not stable enough. The idea is to use pure Delaunay triangulator to produce an initial Delaunay triangulation and later on to constrain it to the line segments (in other words, to split the processing into two steps, each of which is stable enough and the combination of them works much faster). The algorithm also minimizes the number of the newly introduced triangulation points - new points are added only if any of the line segment's endpoints does not match an existing triangulation point.
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1 Introduction

The Edge Swapping method [11, 12, 13, 14] (also known as Edge Flipping) is a powerful approach which allows conversions between different triangulations of the same set of points.

Some denotes: Let $P_n = \{p_0, \ldots, p_{n-1}\}$ is a set of distinguishable points on the plane. A partition of the convex hull of P_n into a set of non-overlapping triangles $T = \{t_0, \ldots, t_{m-1}\}$ (which fully cover the convex hull) is called triangulation of P_n. The points from P_n are called vertices in terms of the triangulation and the edges of the triangles are called edges of the triangulation. An edge is swappable if it is contained in the boundary $B_{t_i t_j} = t_i \bigcup t_j$ formed by two adjacent triangles t_i and t_j of T which can happen only if this boundary $B_{t_i t_j}$ is a convex quadrilateral (see figure 1). By edge swapping is meant the operation of replacing the existing edge (the existing diagonal in $B_{t_i t_j}$) by the other diagonal of $B_{t_i t_j}$ (see figure 2).

Sleator et al. [14] proved that a transformation of different triangulations of same convex polygon into each other by using edge swapping operations is always possible. Their more important for us result is that if there is at least one edge in a triangulation T1 which can be swapped (is swappable) so that an edge of triangulation T2 is produced, then there is a sequence of edge swapping operations that transforms T1 into T2. Cai and Hirsch [15] extended this results for the triangulations of planar surfaces and showed that the complexity of the edge swapping converting between two triangulations of same set of N points is at most $O(N^2)$. Later on Hurtado et al.