

INITIAL SIMPLICIAL COMPLEXES ASSOCIATED TO SOME TORIC VARIETIES*

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Abstract

We study the triangulation of the initial simplicial complex arising from the toric deformation of the Grassmann variety $\mathbb{G}(1, n)$ and the Hankel variety $H(1, n)$, for $n = 3, 4$.

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1 Introduction

Simplicial complexes on a finite set of vertices arise in different ways in commutative algebra, as described in ([14], [16]). In particular, if we consider a graded ideal I of the polynomial ring $S = K[x_1, \dots, x_n]$, K any field, and a total order on the monomials of S , let $in_{\prec}(I)$ be the initial ideal of I . The ideal $J = \sqrt{(in_{\prec}(I))}$ is a monomial squarefree ideal of S and it defines a simplicial complex Δ , whose the Stanley-Reisner ideal is J . If we have a semigroup ring $R \subset S$, generated on \mathbb{K} by monomials of S its definition ideal is a binomial ideal $I_{\mathcal{A}}$ and $J_{\mathcal{A}} = \sqrt{(in_{\prec}(I_{\mathcal{A}}))}$ defines a simplicial complex $\Delta_{\mathcal{A}}$, where \mathcal{A} is the set of lattice points generating the semigroup subtended by R . We call such a simplicial complex the simplicial complex arising from the semigroup ring R . In this paper we are interested to semigroup

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