COMBINATORIAL INTERPRETATIONS OF q-VANDERMONDE'S IDENTITIES[*]

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Abstract

The restricted partitions in which the largest part is less than or equal to N and the number of parts is less than or equal to k were investigated by Andrews in [1]. In this paper, these restricted partitions are extended to the partitions into parts of two kinds. New combinatorial identities are discovered and proved in this way exploring the relationships between Gaussian polynomials and the elementary symmetric polynomials. Combinatorial interpretations of q-Vandermonde's identities are presented in this context.

Keywords: integer partitions; restricted partitions; Gaussian polynomials; q-Vandermonde identities

MSC 2010: 05A17, 11P81, 05A19, 11P84

1 Introduction

In number theory and combinatorics, a partition of a positive integer n is a nonincreasing sequence of positive integers whose sum is n. Two sums that differ only in the order of their terms are considered the same partition. The number of partitions of n is given by the partition function $p(n)$. For example, $p(4) = 5$ because the five partitions of 4 are:

$$4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1. \tag{1}$$