HIGHER-ORDER DIFFERENCES AND HIGHER-ORDER PARTIAL SUMS OF EULER'S PARTITION FUNCTION *

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Dedicated to Professor Mihail Megan
on the occasion of his 70th anniversary

Abstract

We provide generalizations for Euler's recurrence relation for the partition function $p(n)$ and the recurrence relation for the partial sums of the partition function $p(n)$. As a corollary, we derive an infinite family of inequalities for the partition function $p(n)$. We present few infinite families of determinant formulas for: the partition function $p(n)$, the finite differences of the partition function $p(n)$ and the higher-order partial sums of the partition function $p(n)$.

MSC: 05A19, 05A20

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1 Introduction

Let n be a positive integer. In order to indicate that $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_k]$ is a partition of n, i.e.,

$$n = \lambda_1 + \lambda_2 + \cdots + \lambda_k,$$