A THREE-TERM DESCENT CONJUGATE GRADIENT ALGORITHM USING THE MINIMIZATION OF THE TWO-PARAMETRIC QUADRATIC MODEL FOR LARGE-SCALE UNCONSTRAINED OPTIMIZATION

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Abstract. *A three-term descent conjugate gradient algorithm is presented. The algorithm is obtained by minimizing the two-parameter quadratic model of the objective function in which the symmetrical approximation of the Hessian matrix satisfies the general quasi-Newton equation. Using the general quasi-Newton equation the search direction includes a parameter ω which is determined by the formal equality between the search direction used in the suggested algorithm and the Newton direction. It is proved that the best value of this parameter is $\omega = 1$. The direction satisfies both the descent and the conjugacy conditions. The new approximation of the minimum is obtained by the general Wolfe line search using by now a standard acceleration technique. Under standard assumptions, both for uniformly convex functions and for general nonlinear functions, the global convergence of the algorithm is proved. The numerical experiments using a collection of 800 large-scale unconstrained optimization test problems of different complexity show that using these ingredients we get a search direction able to define a very efficient and robust three-term conjugate gradient algorithm. Numerical comparison of this algorithm versus well known conjugate gradient algorithms ASCALCG, CONMIN, AHYBRIDM, CG-DESCENT, THREECG and TTCG as well as the limited memory quasi-Newton algorithm LBFGS (m=5) and the truncated Newton TN show that our algorithm is more efficient and more robust.*

Keywords: Large scale unconstrained optimization, Two parameters quadratic model, Generalized secant equation, Conjugate gradient algorithms, Numerical comparisons

1. Introduction

For solving large-scale unconstrained optimization problems

$$\min_{x \in R^n} f(x), \tag{1.1}$$

where $f : R^n \to R$ is a continuously differentiable function, supposed to be bounded from below, starting from an initial guess $x_0 \in R^n$, a three-term conjugate gradient method we want to develop in this paper, generates the sequence $\{x_k\}$ as:

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