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THE CALCULUS OF ULTRASONIC RESONATORS, USED TO PRODUCES VIBRATION HAVING HIGH ENERGY, BY APPLYING PROPAGATION RELATIONS THROUGH ELASTIC MATERIAL

Grigore Liviu ODOBESCU[1]

Rezumat. *Această lucrare prezintă o metodă pentru determinarea tensiunilor mecanice şi variaţia lor în cazul sistemelor de transmitere şi amplificare a vibraţiilor mecanice produse de transductoarele piezoelectrice de mare intensitate. Metoda permite rapid vizualizarea a 3 parametrii: forma de variaţie a secţiunii, forma de variaţie a amplitudinii de vibraţie (care se poate verifica prin măsurători cu un accelerometru, cu un microscop sau cu metoda descrisă în [2]) şi forma de variaţie a tensiunilor mecanice pentru orice variaţie de secţiune care poate fi descrisă printr-o ecuaţie matematică.*

Abstract. *This paper presents a method for the determination of mechanical tensions and their variations in the case of transmitting and amplifying systems for mechanical vibrations produces by high intensity piezoelectric transducers. The method quickly enables the viewing of 3 parameters: the shape of section variation, the shape of the amplitude vibration variation (which may be checked by measuring with an accelerometer, with a microscope or with the method described in* [2]*) and the shape of the mechanical tensions variation for any section variation which may be described through a mathematical equation.*

Key words: propagation equations, ultrasound vibration

1. Introduction

The method enables the drawing of the three variation shapes for any sections variation which can be mathematically described and for more complex systems formed by systems which transmit and amplify the ultrasonic energy vibrations.

2. Methods

The propagation relation of longitudinal plane waves through bars with variable sections is given by [1]:

$$\frac{d}{dx}\left[A(x)\cdot\frac{d\zeta}{dx}\right]=\frac{A(x)}{c^2}\cdot\frac{d^2\zeta}{dt^2}\text{ ; where }c^2=\frac{E}{\rho} \tag{1}$$

[1]Eng. Ph.D. Institute of Solid Mechanics , Romanian Academy, Constantin Mille nr.15 , 70701 Bucharest , Romania, e-mail: g_odobescu@yahoo.com.

with condition: $\left(\frac{d\zeta}{dx}\right)_{x=0} = \left(\frac{d\zeta}{dx}\right)_{x=l}$ - the vibration amplitude is maximum at the ends.

Where: - $\zeta(x,t)$ -the vibration amplitude in Ox direction;
- $A(x)$ - represents the transversal section area at x distance;
- c- represents the propagation velocity of waves through the bar;
- E - represents the resilience module;
- ρ - represents the density of the bar material.

We have solved this equation and we have calculated the following parameters:

→ the vibration amplitude in Ox direction:

$$\zeta^1(x,t) = X^1(x) \cdot \cos \omega_1 t \quad \text{or} \quad \zeta^1(x,0) = X^1(x)$$

→ the amplification of acoustic chain: $G = \frac{\zeta^1(l,0)}{\zeta^1(0,0)}$

→ the ratio of the areas at the two ends: $\frac{A(0)}{A(l)}$

→ if we name X_0=q the distance where the vibration amplitude $\zeta(x,t)$ is null, we have: $\zeta^1(q,0) = 0$

The resultant X_0 represents the point of rigid catch of the acoustic chain without affecting its functioning.

→ the vibration amplitudes at the two ends: $\zeta^1(0,0)$ and $\zeta^1(l,0)$.

→ the variation curve of mechanical tensions: T_m.

We have the relation $T_m = \rho \cdot c \cdot v_m$,

Where: - ρ - is the density of the bar material;
- c - is the ultrasound velocity through the bar material;
- v_m - is the velocity of the vibrating particle.

The movement of the ultrasonic wave through the bar material follows a sinusoidal law given by relation: $u(t) = U_m \cdot \sin \omega t$

where: - $u(t)$ - is the momentary amplitude of vibration;
- U_m - is the maximum amplitude of vibration;
- $\omega = 2\pi f$, f is the frequency of the ultrasonic vibrations.

The velocity of the particles, the derivative of the movement will be given by relation $v(t) = \frac{du(t)}{dt} = U_m \cdot \omega \cdot \cos \omega t$ and the acceleration of the particles which are in an ultrasonic vibration field will be: $a(t) = \frac{dv(t)}{dt} = -U_m \cdot \omega^2 \cdot \sin \omega t$.

From the above relations results the proportionality between the mechanical tension T_m, the velocity of particles and the derivative of vibration amplitudes. So, we have: $T_m \sim v_m \sim \frac{d}{dx}\left[\zeta^1(x,0)\right]$.

3. Results.

The resonance bodies is formed from bars with diminishing variable sections (exponential, by steps, in the shape of a truncated cone) The acoustic chain is formed from two or more resonance bodies which are fastened together by a screw and it is excited by a piezoelectric transducer with a resonant frequency of ultrasonic propagation through the material bar. In fig. 1 it is presented a resonance body with variable section by steps. In fig. 2 it is presented a resonance body with variable section by steps connected by circular radius r_{ac}. In fig. 3 it is presented a resonance body with variable section by step connected by exponential radius. In fig. 4 it is presented a resonance body with variable section in the shape of a truncated cone. With the help of the calculation program and a presented method we have obtained the following conclusions:

→ measured the resonance frequency of these resonance bodies by using the echo method of an ultrasound which has a variable frequency. We determined the maximum echo;

→ determined the propagation velocity, through calculation, for these resonance bodies by $v = \lambda \cdot f$ and compared with the geometrical dimensions for make a verification of this method with experimental results.

→ determined the shapes of the vibration amplitude variation in Ox direction given by ζ^1 curve. Notice the null vibration point where it is possible to catch the mechanical ensemble.

→ determined the areas of the resonance bodies sections which are given by expression $\sqrt{\frac{A(z_p)}{\pi}} \cdot 10^2$;

→ determined the points with maximum demand; experiments have also shown that because high level of mechanical tension, when acoustic systems are subjected to high ultrasonic fields they break or fissure along the points with maximum demand.

→ determined the influence of the variation of geometrical dimensions on the shapes of mechanical tensions and on the position of the null vibration point.

→ determined the mechanical tensions in the resonance bodies which are given by $(\Delta\zeta_{1p}) \cdot 10^{-2}$, where $\Delta\zeta 1(x,t) = \frac{d}{dx}\zeta_1(x,t)$.

We observe that the mechanical tension is at its maximum immediately after the section step jump takes place, while its level is mitigated when the passage between two sections happens through a connection radius.

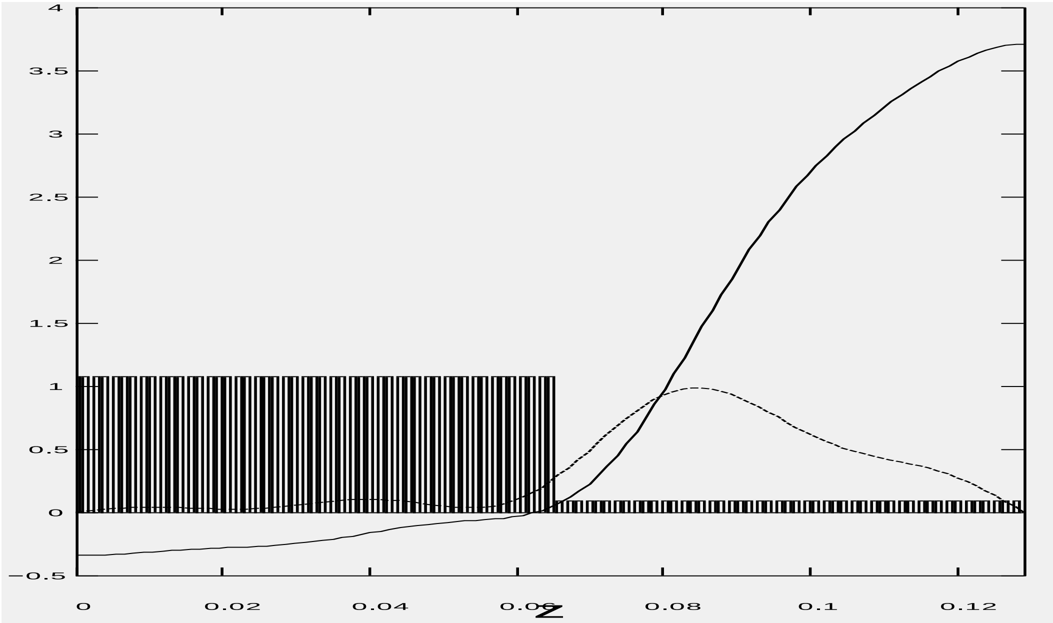

Fig. 1. Resonance body with variable section by steps.
-- The variation of vibration amplitude of resonance body.
The variation of resonance body section.
- -The variation of mechanical tensions in resonance body.

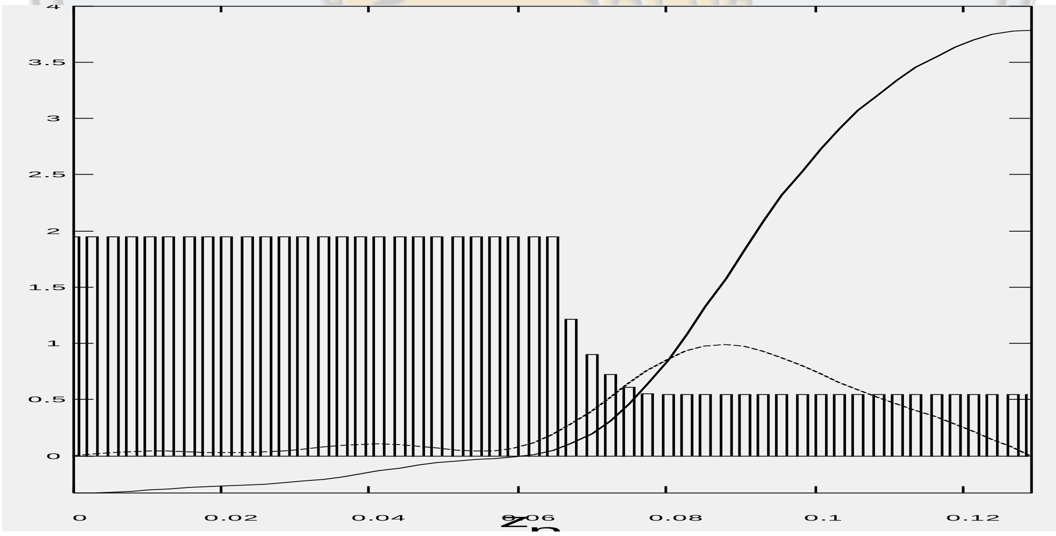

Fig. 2. Resonance body with variable section by steps connected by circular radius r_{ac}.
----The variation of vibration amplitude of resonance body.
The variation of resonance body section.
- - - The variation of mechanical tensions in resonance body.

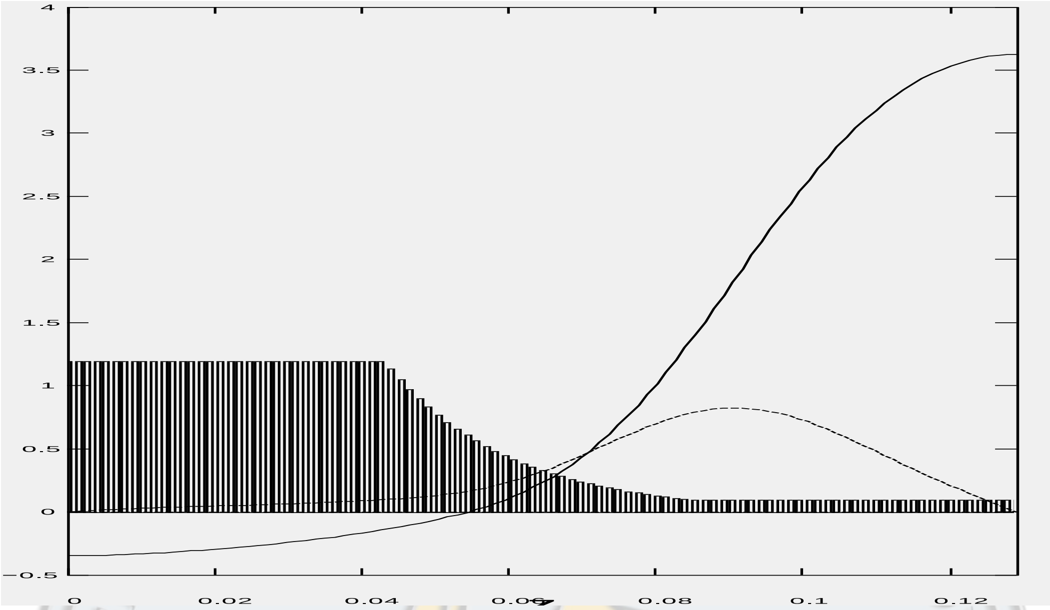

Fig. 3. Resonance body with variable section by step connected by exponential radius.
---- The variation of vibration amplitude of resonance body.
The variation of resonance body section.
- - - The variation of mechanical tensions in resonance body.

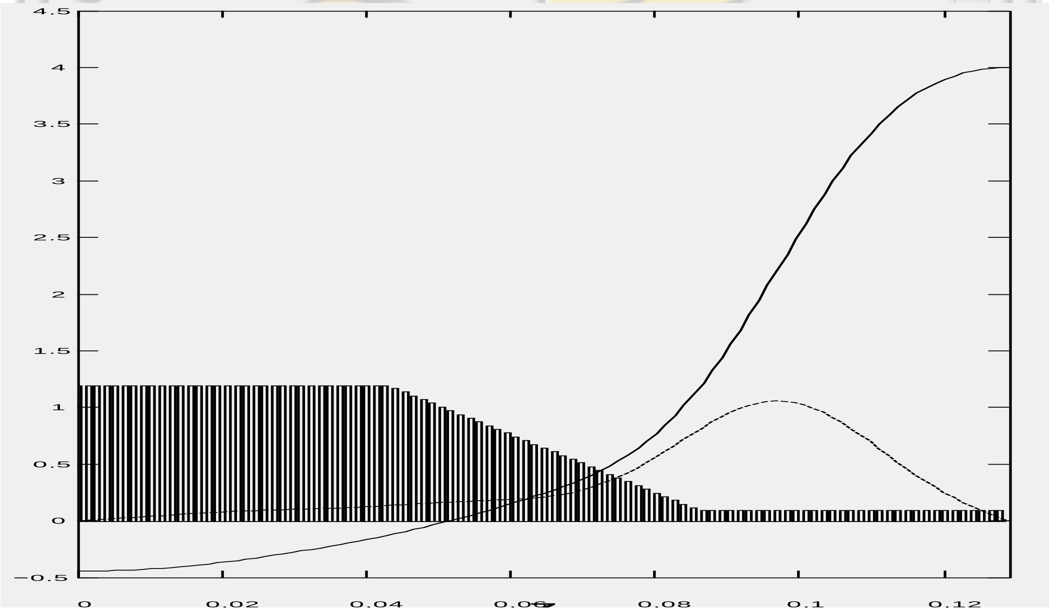

Fig. 4. Resonance body with variable section in the shape of a truncated cone.
---- The variation of vibration amplitude of resonance body.
The variation of resonance body section.
- - - The variation of mechanical tensions in resonance body.

Conclusions

With the method presented in this paper, I have realized and verified the acoustic chain systems. The method has the following advantages:

a) allows to quickly determine the shapes of the variation for: the mechanical tension, the vibration amplitude, the sections areas;

b) allows to follow the influence of different factors on these variation shapes: the material that makes up the acoustic chain, the connection radius and its variation law, the length of elements;

c) allows to expand to more complex shapes of the acoustic chain system with a length of $n \cdot \lambda/2$; n=1, 2, 3.

R E F E R E N C E S

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