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SHORT OPTICAL PULSES EVOLUTION IN NONLINEAR DIELECTRIC THIN LAYERS

Mihai C. PISCUREANU[1], Vicenţiu CONSTANTIN[2]

Rezumat. *În lucrare se analizează propagarea impulsurilor temporale intr-un strat dielectric subţire neliniar constând dintr-un material dispersiv neomogen, în prezenţa efectelor de rezonanţă. Am aplicat o metoda perturbaţională şi cu ajutorul acesteia am obţinut soluţii solitonice de tip anvelopă, stabilind astfel condiţiile de existenţă a solitonilor strălucitori şi respectiv întunecaţi cu ajutorul coeficienţilor ecuaţiei Schrödinger neliniare şi al relaţiei de dispersie a undei cuasiplane. Aceste rezultate teoretice au fost aplicate în cazul unei structuri dielectrice planare cu* SiO_2*. În acest caz am arătat că în vecinătatea frecvenţei de rezonanţă şi a frecvenţei de tăiere superioare solitonii încetează de a mai fi impulsuri scurte temporal.*

Abstract. *In this paper we have analyzed the propagation of short temporal pulses in a nonlinear dielectric thin layer consisting of an inhomogeneous dispersive material in the presence of the resonance effects. We used a perturbational method and we obtained envelope solitons. Thus, we have established the existence conditions for the bright and dark solitons from the nonlinear Schrödinger equation coefficients by using a cuasiplanar wave dispersion relation. The theoretical results have been applied in the case of a typical* SiO_2 *slab. Consequently, we have shown that in the vicinity of the resonance frequency and the upper cut-off frequency, the solitons cease to be temporally sharp pulses.*

Keywords: Solitons, planar waveguides, nonlinear Schrödinger equation, perturbations theory

1. Introduction

In this paper we analyze the propagation of the pulses in a dispersive Kerr-type medium, represented by an infinitely long dielectric, without losses and with a parabolic profile of the refractive index. Our analysis has the following stages:

a. The effects of the transversal inhomogeneities are decoupled from the longitudinal propagation in the planar dielectric medium. This is achieved by an averaging method, thus obtaining in the monomodal case a nonlinear wave equation describing the longitudinal propagation of the pulses.

b. We consider all the harmonics generated by the nonlinearity by expanding the unknown function in a power series of the ε parameter ($\varepsilon \ll 1$).

[1] University "Politehnica" of Bucharest, Chair of Physics 2, Splaiul Independenţei 313, Bucharest, 77206, Romania (piscureanu@physics.pub.ro).
[2] "Mircea cel Batrân" National Collegium, Carol I, 41, Râmnicu Vâlcea, România.

We analyze the case of the temporally short pulses, for which the parameter ε is given by the ratio between the spectral width $\Delta\omega$ and the carrier angular frequency ω.

c. We establish the integrability conditions for the order of ε, thus obtaining the corresponding group velocity expression and the dispersion relations.

From the integrability conditions corresponding to $O\left(\varepsilon^3\right)$ we establish the nonlinear Schrödinger equation (NLS).

By means of NLS we establish the conditions leading to the generation of bright and dark solitons in the presence of the medium dispersion.

2. Theoretical model

a. The refractive index $n(x)$, where x is the transverse coordinate, is given by:

$$n^2(x) = n_1^2\left[1 - 2\Delta\left(\frac{x}{d}\right)^2\right] + K_e\left|\overline{E}\right|^2 \tag{1}$$

where n_1 is the linear refractive index for $x=0$, $\Delta = (n_1 - n_0)/n_1$ is the relative variation of the film refractive index with respect to that of the coating; K_e is the Kerr coefficient and $\overline{E}$ is the electric field intensity vector.

Expression (1) is valid as long as the resonant effects can be neglected.

The resonance effects denoted by $\Delta\overline{P}_L$ can be described as a linear contribution to the polarization vector $\overline{P}_L = \varepsilon_0\left[n^2(x) - 1\right]\overline{E}$, and are given by a convolution integral:

$$\Delta\overline{P}_L = \varepsilon_0 \cdot \int_{-\infty}^{z} dz' \int_{-\infty}^{t} dt' \,\delta(z - z') \cdot b(z - z'; t - t')\overline{E}(x, z', t') \tag{2}$$

ε_0 is the dielectric constant of the vacuum, $\delta(\bullet)$ is the delta function, expressing the spatial homogeneity along z-axis, and $b(z,t)$ is the anomalous part of the response function of the dispersive medium.

Considering the even TE mode case with $\overline{E} = \overline{j}\overline{E}_y(x,z,t)$ and using the Maxwell equations, we get the following wave equation:

$$\frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_y}{\partial x^2} - \frac{n_1^2}{c^2}\left[1 - 2\Delta\left(\frac{x}{d}\right)^2\right]\frac{\partial^2 E_y}{\partial t^2} - \\ -\left(K_e / c^2\right)\frac{\partial^2}{\partial t^2}\left(E_y\left|E_y\right|^2\right) - \frac{1}{c^2}\frac{\partial^2(\Delta P_L)}{\partial t^2} = 0 \tag{3}$$

where $\Delta P_L = \left|\Delta \bar{P}_L\right|$.

In the linear case and in the absence of the resonance effects $\left(K_e = 0, \Delta P_L = 0\right)$, the solution of equation (3) can be expressed as Hermite polynomials depending of the transverse coordinate x. For the linear monomode case, E_y becomes:

$$E_y(x,z,t) = E_0 \exp\left(-x^2 / w^2\right) \cdot \exp\left[i(\omega_0 t - \beta_0 z)\right] \tag{4}$$

where E_0 is the normalizing constant, ω_0 is the angular frequency, w is the beam radius and β_0 is the wave vector in the considered medium.

In the nonlinear case, the effects of the transverse inhomogeneities are taken into account by averaging. Thus, taking $E_y(x,z,t)$ as:

$$E_y(x,z,t) = U(z,t) \cdot \exp\left(-x^2 / w^2\right) \tag{5}$$

after applying an averaging method, we get the following onedimensional nonlinear wave equation:

$$\frac{\partial^2 U}{\partial z^2} - \frac{U}{w^2} - \frac{n_1^2}{c^2}\left(1 - \frac{\Delta w^2}{2d^2}\right)\frac{\partial^2 U}{\partial t^2} - \frac{K_e}{\sqrt{2}\cdot c^2}\frac{\partial^2}{\partial t^2}\left(U\left|U\right|^2\right) = \\ = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{z} dz' \int_{-\infty}^{t} dt' \delta(z - z') \cdot b(z - z', t - t') \cdot U(z', t') \tag{6}$$

b. The perturbational method

Suppose that the unknown function $U(z,t)$ can be expanded by the powers of the ε parameter $(\varepsilon << 1)$:

$$U(z,t) = \sum_{l=-\infty}^{\infty}\sum_{a=1}^{\infty} \varepsilon^a U_l^a(\xi, \tau) \cdot \exp[-il(kz - \omega t] \tag{7}$$

where k is the wave number, ω is the angular frequency and the summation is made for all the harmonics generated by the nonlinear response of the polarization. U_l^a is the envelope of the l-th harmonics of the approximation of order a, slowly varying with z and t.

The variables ξ and τ are defined by the following scaling relations:

$$\xi = \varepsilon^2 z, \quad \tau = \varepsilon\left(t - \frac{z}{v_g}\right) \tag{8}$$

where v_g is the group velocity.

From relations (6) – (8) we get the following equation for the l-th component $U_l^a(\xi,\tau)$:

$$\sum_{a=1}^{\infty}\varepsilon^a\left\{\left(\varepsilon^2\frac{\partial}{\partial\xi}-\frac{\varepsilon}{v_g}-ilk\right)^2 U_l^a-\frac{1}{w^2}U_l^a-\frac{n_1^2}{c^2}(1-\rho)\left(\varepsilon\frac{\partial}{\partial\tau}+il\omega\right)^2 U_l^a-\right.$$
$$-\frac{K_e}{\sqrt{2}\cdot c^2}\sum_{\substack{n=-\infty\\ m=-\infty}}^{\infty}\sum_{\substack{b=1\\ c=1}}^{\infty}\varepsilon^{b+c}\left(\varepsilon\frac{\partial}{\partial\tau}+il\omega\right)^2\left(U_n^a\cdot U_m^b\cdot U_{l-n-m}^c\right)-$$
$$\left.-\frac{1}{c^2}\sum_{\substack{n=0\\ m=0}}^{\infty}\frac{(-i)^{n-m}}{n!m!}\varepsilon^{n+2m}\left(\varepsilon\frac{\partial}{\partial\tau}+il\omega\right)^2\left(\frac{\partial^{n+m}U_l^a}{\partial\tau^n\partial\xi^m}\right)\cdot\left(\frac{\partial^{n+m}b_{\omega,k}}{\partial\omega^n\partial k^m}\right)_{\substack{\omega=l\omega\\ k=lk}}\right\}$$
$$. \tag{9}$$

where $p = \left(\sqrt{\Delta/2}\right)/(k_0 n_1 d)$ and $b_{\omega,k}$ is the Laplace transform of $b(z,t)$ function).

In order to separate different orders of ε, we introduce the following operator:

$$L = -k^2 + \frac{\omega^2}{c^2}\left[n_1^2(1-p)+b_\omega\right]-\frac{1}{w^2} \tag{10}$$

where $b_\omega = b_{\omega,k}$.

Finally we get:

- third order:

$$\left\{-\frac{1}{2}\frac{\partial^2 L}{\partial\omega^2}+\left(\frac{1}{V_g^2}-\left(\frac{\partial k}{\partial\omega}\right)^2\right)-k\frac{\partial^2 k}{\partial\omega^2}\right\}\frac{\partial^2 U_l^1}{\partial\tau^2}+$$
$$+i\left\{\frac{\partial}{\partial k}\left(L-\frac{\omega^2}{c^2}\left[n_1^2(1-p)+b_\omega\right]\right)+\frac{\omega^2}{c^2}\frac{\partial b_\omega}{\partial k}\right\}\cdot\frac{\partial U_l^1}{\partial\xi}-$$
$$-i\left\{\frac{\partial L}{\partial\omega}+\left(\frac{1}{V_g}-\frac{\partial k}{\partial\omega}\right)\frac{\partial}{\partial k}\left(L-\frac{\omega^2}{c^2}\cdot\left[n_1^2(1-p)+b_\omega\right]\right)\right\}\frac{\partial U_l^2}{\partial\tau}+LU_l^3+$$
$$+\omega^2\frac{K_e}{\sqrt{2}c^2}\sum_{\substack{n=-\infty\\ m=-\infty}}^{\infty}U_n^1U_m^1U_{l-n-m}^1=0 \tag{11}$$

c. Integrability conditions

From (11) we obtain the integrability conditions for $O\left(\varepsilon^3\right)$:

$$L=0 \tag{12}$$

$$\frac{\partial L}{\partial\omega}+\left(\frac{1}{v_g}-\frac{\partial k}{\partial\omega}\right)\frac{\partial}{\partial k}\left(L-\frac{\omega^2}{c^2}\left[n_1^2(1-p)+b_\omega\right]\right)=0 \tag{13}$$

$$\left\{-\frac{1}{2}\frac{\partial^2 L}{\partial\omega^2}-k\frac{\partial^2 k}{\partial\omega^2}\right\}\frac{\partial^2 U_l^1}{\partial\tau^2}+i\left\{\frac{\partial}{\partial k}\left(L-\frac{\omega^2}{c^2}\left[n_1^2(1-p)+b_\omega\right]\right)+\right.$$
$$\left.+\frac{\omega^2}{c^2}\frac{\partial b_\omega}{\partial k}\right\}\cdot\frac{\partial U_l^1}{\partial\xi}+\omega^2\frac{K_e}{\sqrt{2}\cdot c^2}\sum_{\substack{n=-\infty\\ m=-\infty}}^{\infty}U_n^1U_m^1U_{l-n-m}^1=0 \tag{14}$$

Equation (14) is a nonlinear Schrödinger-like equation, whose analysis is useful in establishing the conditions for the generation of bright and dark solitons.

By means of relations (10) and (14) for one mode propagation, we get:

$$i\left(\frac{\omega^2}{c^2}\frac{\partial b_\omega}{\partial k}-2k\right)\frac{\partial U_1^1}{\partial\xi}-k\frac{\partial^2 k}{\partial\omega^2}\cdot\frac{\partial^2 U_1^1}{\partial\tau^2}+\frac{3K_e\omega^2}{\sqrt{2}\cdot c^2}U_1^1\left|U_1^1\right|^2=0 \tag{15}$$

When the resonance effects can be neglected, $(b_\omega = 0)$, equation (15) becomes:

$$i\frac{\partial U_1^1}{\partial \xi} - \frac{1}{2}\frac{\partial^2 k}{\partial \omega^2}\cdot\frac{\partial^2 U_1^1}{\partial \tau^2} + \nu U_1^1 \left|U_1^1\right|^2 = 0 \tag{16}$$

where:

$$\nu = \frac{3K_e \omega}{2\sqrt{2}c^2}\cdot \mathrm{v}_f \tag{17}$$

The relation (15) can be written under the form:

$$i\frac{\partial U}{\partial X} = -\frac{1}{2}\frac{K\dfrac{\partial^2 K}{\partial \Omega^2}}{K - \dfrac{\Omega^2}{2}\cdot\dfrac{\partial b_\omega}{\partial K}}\cdot\frac{\partial^2 U}{\partial T^2} + \frac{3KeU_0^2}{\sqrt{2}c^2}\cdot\frac{\Omega^2}{K - \dfrac{\Omega^2}{2}\cdot\dfrac{\partial b_\omega}{\partial K}} U|U|^2 \tag{18}$$

where

$$\omega = \Omega\omega_e, \quad k = \frac{\omega_e}{c}\cdot K, \quad U_1^1 = U_0 U, \quad \xi = X\cdot\frac{c}{\omega_e}, \quad \tau = T\cdot\omega_e^{-1} \tag{19}$$

ω_e is the resonance frequency of the function b_ω, and U_0 is the initial amplitude of the pulse.

From relation (18) we can establish the conditions for the generation of bright and dark solitons, respectively.

In order to characterize the pulse width we introduce the following parameter:

$$\Omega_0 = \Omega\left|K\frac{\partial^2 K}{\partial \Omega^2}\right|^{-1/2}\cdot \mathrm{sign}\left(K\frac{\partial^2 K}{\partial \Omega^2}\right) \tag{20}$$

named characteristic frequency, and being correlated to the temporal width of the pulse.

In the absence of the resonance effects $(b_\omega = 0)$ because ν is always positive and

$$\frac{1}{2}\frac{\partial^2 k}{\partial \omega^2} = -\frac{1}{2}\frac{1}{v_g\omega}\cdot\left(\frac{v_f}{v_g}-1\right) \tag{21}$$

is negative $(v_f > v_g > 0)$, it results that the equation (16) has only bright solitons solutions.

3. Analysis of the bright and dark solitons generation conditions in a planar dielectrical structure

Suppose that the function $b(z,t)$ is given by:

$$b(z,t) = b(t) = \frac{\omega_p^2}{\omega_e}\sin\omega_e t \tag{22}$$

and consequently the function $b_{\omega,k}$ is of the form:

$$b_{\omega,k} = b_\omega = \frac{\omega_p^2}{\omega_e^2 - \omega^2} \tag{23}$$

In this case, b_ω corresponds to the real part of the electric susceptibility and ω_e and ω_p are the electric dipole resonance frequency and the electronic plasma frequency, respectively.

By means of the expression of b_ω, we can obtain the dispersion relation L=0 under the form:

$$c^2k^2 + \frac{c^2}{w^2} = \omega^2\left(n_1^2(1-p) + \frac{\omega_p^2}{\omega_e^2 - \omega^2}\right) \tag{24}$$

In order to calculate the characteristic frequency Ω_0 and establish its sign we use the relation (24) under the form:

$$\frac{K^2}{n_1^2(1-p)} = \frac{\left(\Omega_s^2 - \Omega^2\right)\left(\Omega^2 - \Omega_i^2\right)}{\left(1-\Omega^2\right)} \tag{25}$$

where Ω_s and Ω_i are the upper and lower cut-off frequencies respectively:

$$\Omega_s^2 = 1 + \frac{\omega_p^2}{2\omega_e^2 n_1^2(1-p)}$$
$$\Omega_i^2 = \frac{c^2}{\omega_e^2 w^2 n_1^2(1-p)} \tag{26}$$

For the cuasiplanar wave approximation, the propagation is possible when $\Omega_i \le \Omega \le 1$ or $\Omega_s \le \Omega$.

When resonance is neglected, the waves propagate when $\Omega_i \le \Omega$, the dispersion relation becoming:

$$K(\Omega) = \pm n_1 (1-p)^{1/2} \left(\Omega^2 - \Omega_i^2\right)^{1/2} \qquad (27)$$

and due to the fact the upper cut-off frequency becomes equal to 1, we get:

$$\frac{1}{\Omega_0^2} = n_1^2 (1-p) \left| -\frac{\Omega_i^2}{\Omega^2 - \Omega_i^2} \right|. \qquad (28)$$

Because Ω_0 is always negative it follows that equation (18) will have only bright soliton-like solutions.

In figures 1 and 2 the characteristic frequency Ω_0 is represented as a function of the normalized frequency Ω and of the normalized wave vector *K* respectively.

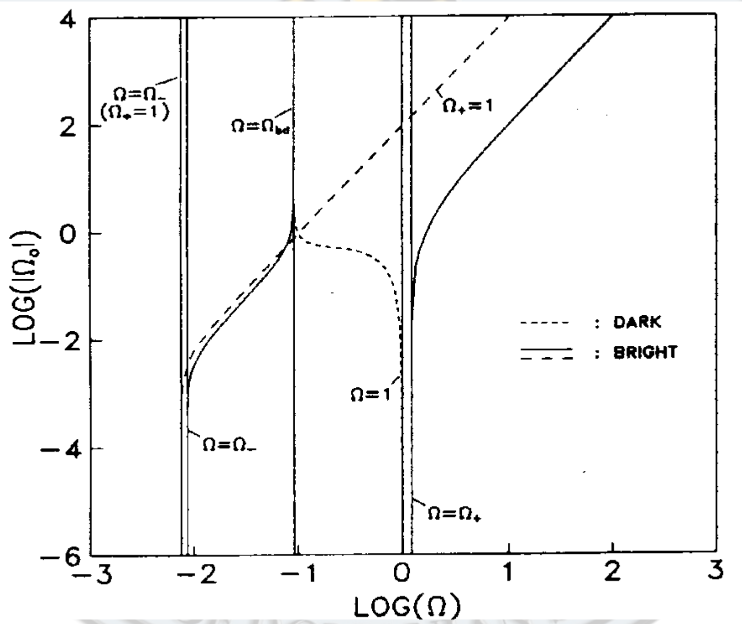

Fig. 1. Characteristic frequency Ω_0 dependence on normalized frequency Ω for bright and dark solitons case.

The positive values (dark solitons) and the negative values (bright solitons) of Ω_0 are shown by the fine dashed lines and by solid lines respectively.

The long dashed lines refer to the case of the resonanceless dispersion; in this case the absolute value of Ω_0 behaves like a linear function of Ω and of *K* respectively for values of Ω much larger than the cut-off frequency Ω_i $(\Omega_s = 1)$.

If the resonance effects are present, then we must take into consideration the upper cut-off frequency Ω_s $(\Omega_s > 1)$ and the resonance frequency $\Omega = 1$.

The graphs have been drawn for planar SiO_2 media, the parameters being properly chosen to permit the propagation of a single mode, namely: dielectric film thickness $d = 2\ \mu\text{m}$; the relative variation of the core refractive index with respect to that of the cover, $\Delta = 0{,}01$; the vacuum wavelength, $2\pi/k_0 = 0{,}588\ \mu\text{m}$.

The electric dipole resonance frequency is 20358 THz and the electron plasma frequency is 21321 THz.

The corresponding normalized cut-off frequencies are $\Omega_i = 8{,}62 \cdot 10^{-3}$ and $\Omega_s = 1{,}23$ respectively.

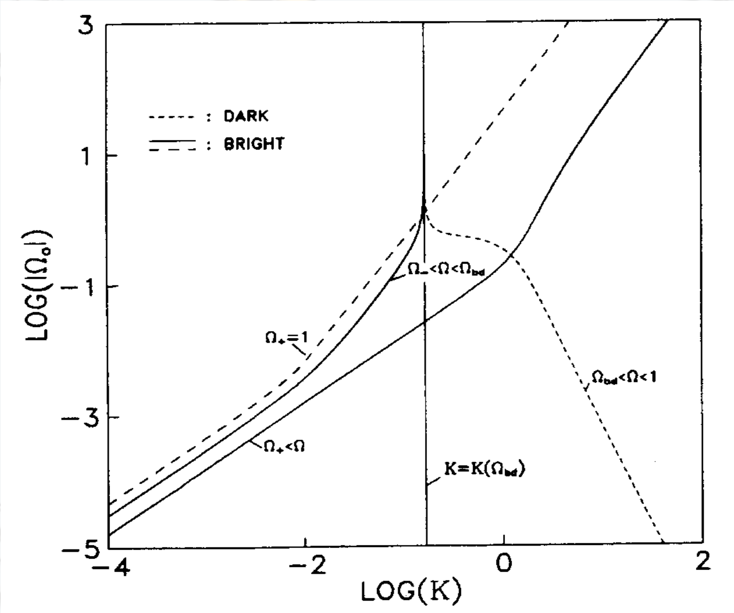

Fig. 2. Characteristic frequency Ω_0 dependence on normalized wave vector K for bright and dark solitons case.

From figures 1 and 2 we noticed the existence of a normalized transition frequency Ω_t, around which the characteristic parameter Ω_0 has high values, which implies the fact that solitons with short temporal width propagate in the vicinity of that frequency.

It is significant the fact that nature of solitons changes when passing through the transition frequency, namely from bright solitons to dark ones.

Dark solitons propagation takes place within the frequency range between the transition frequency Ω_t and the resonance one $\Omega = 1$.

The temporal width of the temporal "kink"-like solitons increases far from the transition frequency Ω_t and rapidly decreases when Ω approaches the resonance value $\Omega = 1$.

On the other side, the bright solitons have narrow width only for frequencies much larger than the cut-off frequencies Ω_i and Ω_s.

Conclusions

It has been shown that in the absence of the resonance effects in the planar waveguide material only bright soliton-like solutions exist.

For short temporal pulses a bright-to-dark solitons transition occurs at a critical frequency Ω_t.

However, at the frequency Ω_s $(\Omega_s < \Omega)$ or at resonance $(\Omega \leq 1)$ the bright and dark soliton solutions are not short temporal pulses any longer, due to the rapid decrease of their characteristic frequency Ω_0.

In conclusion, the existence of a bright-to-dark solitons transition at the frequency Ω_t, lying between the lower cut-off frequency Ω_i and the upper one Ω_s is a characteristic feature of the short temporal pulses evolution and it is due to the presence of the resonant dispersive properties of the dielectric medium.

R E F E R E N C E S

[1] Y. Kodama, J. Stat. Phys., **39**, 597-614, (1985).

[2] J. Satsuma and N. Yajima, Progr. Theor. Phys. (Japan), suppl. 55, pp. 284-306, (1974).

[3] G. L. Lamb Jr., *Elements of Soliton Theory*, New York: Wiley, (1980).

[4] C. M. Piscureanu, *Modele solitonice în optica neliniară cu aplicaţii,* Litografia UPB, (1998).

[5] Andrei Petrescu; Andreea Rodica Sterian; Paul Sterian, *Solitons Propagation in Optical Fibers Computer Experiments for Students Training,* lecture Notes in Computer Science, Springer Berlin/ Heidelberg, Volume 4705/2007, pp 450-461 (Book: Computational Science and its Applications - ICCSA 2007).

[6] Valerică Ninulescu, Andreea Rodica Sterian, *Dynamics of a Two – Level Medium Under the Action of Short Optical Pulses*, Computational Science and Its Applications – ICCSA – 2005, Gervasi et al. (Eds): Lecture Notes in Computer Science (ISI web off Science), LNCS 3482, pp. 635- 642, Springer – Verlag, Berlin, Heidelberg, (2005).

[7] Valerică Ninulescu, Andreea Rodica Sterian, *Nonlinear Phenomena in Erbium – Doped Lasers*, Computational Science and Its Applications – ICCSA – 2005, Gervasi et al. (Eds): Lecture Notes in Computer Science (ISI web off Science), LNCS 3482, pp. 635- 642, Springer – Verlag, Berlin, Heidelberg, (2005).