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COMPUTER SIMULATIONS OF INTERACTION EFFECTS IN SOLITON TRANSMISSION, FOR STUDENTS TRAINING

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Rezumat. *Articolul de faţă îşi propune să prezinte unele rezultate numerice ale simulării propagării tri-solitonilor, bazate pe ecuaţia Körteweg-de Vries (KdV), utilizând (Maple11), un program puternic, ce permite realizarea calculelor numerice, trasarea şi animarea reprezentărilor grafice, operarea cu expresii analitice. Este discutată interacţiunea tri-solitonilor în prezenţa perturbaţiilor, pentru a găsi modalităţile de creştere a capacităţii de transmisie şi de micşorare a ratei erorilor la nivel de bit. Aceste simulări au fost gândite pentru a fi utile atât proiectanţilor sistemelor de transmisii digitale de date şi studenţilor, pentru a sigura o mai bună înţelegere a acestor fenomene. Acest articol extinde unele rezultate obţinute pentru soluţile uni şi bi-solitonice ale equaţiei KdV.*

Abstract. *This paper aims to present some numerical simulations of tri-soliton propagation, based on Körteweg-de Vries (KdV) equation using (Maple11), a powerful program that permits to perform numerical calculations, plot or animate functions and manage analytical expressions. We discuss the tri-soliton interaction in perturbations' presence, in order to find the modalities for raising the transmitting capacity and decreasing the bit-error rate. These simulations are thought to be useful both for the designers working in digital data transmission and students, for better understanding of those phenomena. This article extends some results for the one and two-soliton solutions of KdV equation.*

Keywords: soliton, Körteweg-de Vries equation, computer simulation

1 Introduction

In [1] we will discuss a model for solving *Optoelectronic* problems by means of a computer environment (**Maple10**) which allows both numerical and symbolic solving of a wide range of applications (2D or 3D plots and plots animation).

In this paper we refer to the solitonic solutions of the *Korteweg-de Vries* equation (KdV) and we show some numerical and graphical analysis of the solutions of the KdV equation.

Finally, we present and we discuss some 3D graphics and 2D animated graphs for the tri-solitonic solution of KdV equation.

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2 Solitons in Optical Fibers

As we know, solution of the nonlinear evolution equation is the localized nonlinear wave called soliton ([2], [3]). The existence of such a solution is due to the balance between fiber dispersion and nonlinearity ([4], [5], [6]). This phenomenon occurs in ordinary glass optical fibers, for $\lambda > 1.30$ μm, when the intramodal dispersion is positive ([7]).
In these fibers, the characteristic distance L of soliton propagation is ([3], [8], [9]):

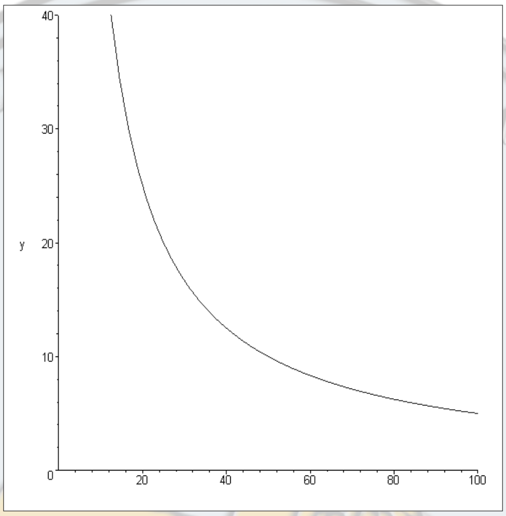

Fig. 1. The characteristic distance of soliton propagation versus the soliton's power

$$L = \frac{500\ \mathrm{W \cdot m}}{P}, \tag{1}$$

where P is the peak power of the soliton. So, for P = 50 mW, L = 10 km; obviously, we may increase L if we use smaller peak power, as seen in fig. 1.
If EDFAs with dispersion-shifted fibers are used, the transmission distance may be 3,000 km at a 40 Gb/s bit rate ([9]).

3. The Fundamental Soliton, as a Solution of the NLS Equation

Wave propagation in an optical fiber with nonlinearity can be described by means of the *nonlinear Schrödinger* equation ([3], [5], [6]):

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 \left(P_{lin} + P_{nonlin}\right)}{\partial t^2}, \tag{2}$$

where P_{nonlin} is the polarization due to the nonlinear Kerr effect ([7], [9]). We suppose E is a wave packet, described by the expression

$$E(z,t) = A(z,t) \times \exp[j(\omega_0 t - \beta_0 z)]. \tag{3}$$

After some calculations, we will get the following expression:

$$\frac{\partial A}{\partial z} + \frac{1}{v_g}\frac{\partial A}{\partial t} - j\frac{\beta}{2}\frac{\partial^2 A}{\partial t^2} = -j\gamma|A|^2 A. \tag{4}$$

Normalizing this expression, we get the canonical form of the NLS equation:

$$\frac{\partial U}{\partial Z} = j\left(\tfrac{1}{2}\frac{\partial^2 U}{\partial T^2} + |U|^2 U\right). \tag{5}$$

The fundamental soliton, described by the equation:

$$U\left(\frac{z}{L_D}, \frac{t - \frac{z}{v_g}}{T_0}\right) = \frac{2\exp\left(j\frac{z}{2L_D}\right)}{\exp\left(\frac{1}{T_0}\left(t - \frac{z}{v_g}\right)\right) + \exp\left(-\frac{1}{T_0}\left(t - \frac{z}{v_g}\right)\right)} \tag{6}$$

$$\text{or } U(Z,T) = \frac{2\exp\left(j\frac{Z}{2}\right)}{\exp(T) + \exp(-T)} \tag{7}$$

verifies the nonlinear Schrödinger equation, as we can see by direct calculations:

because
$$U(T,Z) = \frac{\exp\left(j\frac{Z}{2}\right)}{\tanh(T)} \tag{7'}$$

$$\frac{\partial U}{\partial T} = -U\tanh(T), \ \frac{\partial^2 U}{\partial T^2} = -2|U|^2 U + U \ \text{ and } \ \frac{\partial U}{\partial Z} = j\frac{U}{2}; \tag{8}$$

then
$$\frac{\partial U}{\partial Z} = j\left(\frac{1}{2}\frac{\partial^2 U}{\partial T^2} + |U|^2 U\right). \tag{9}$$

Using Maple11, we can visualize the 3D envelope of this solution (Fig. 2).

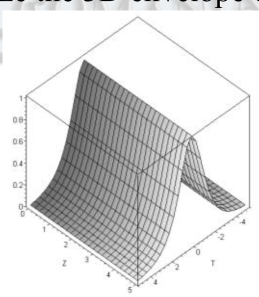

Fig. 2. The 3D plotting of the envelope (7).

4. Korteweg–de Vries Equation Model

Let us consider the Korteweg-de Vries equation (KdV):

$$\frac{\partial \mathrm{u}(x,t)}{\partial t}+6\mathrm{u}(x,t)\frac{\partial \mathrm{u}(x,t)}{\partial x}+\frac{\partial^3 \mathrm{u}(x,t)}{\partial x^3}=0 \tag{10}$$

his one dimensional solitonic solution ([4], [5], [12], [13]), obtained and verified by using Maple11 program:

$$\mathrm{u}_1(x,t)=\frac{2k^2}{\cosh[k(x-4k^2t)]^2}\,. \tag{11}$$

In fact, using Maple11, we can write:

$$KdV := \left(\frac{\partial}{\partial t}\mathrm{u}(x,t)\right)+6\,\mathrm{u}(x,t)\left(\frac{\partial}{\partial x}\mathrm{u}(x,t)\right)+\left(\frac{\partial^3}{\partial x^3}\mathrm{u}(x,t)\right) \tag{12}$$

$$\left(\left(\left(\frac{\partial}{\partial t}\left(\frac{2\,k^2}{\cosh(k\,(-x+4\,k^2\,t))^2}\right)\right)\cosh(k\,(-x+4\,k^2\,t))^2 + 12\,k^2\left(\frac{\partial}{\partial x}\left(\frac{2\,k^2}{\cosh(k\,(-x+4\,k^2\,t))^2}\right)\right) + \left(\frac{\partial^3}{\partial x^3}\left(\frac{2\,k^2}{\cosh(k\,(-x+4\,k^2\,t))^2}\right)\right)\cosh(k\,(-x+4\,k^2\,t))^2\right)\right) \Big/ \cosh(k\,(-x+4\,k^2\,t))^2 \tag{13}$$

so, the solution $u_1(x, t)$ is:

$$\frac{2\,k^2}{\cosh(k\,(-x+4\,k^2\,t))^2} \tag{14}$$

We must observe that, from the solutions of the free particle Schrodinger equation and using Maple11 we can easily obtain the solutions of KdV equation as follows:

$$0 = H_i\,\psi_i \tag{15}$$

by means of the Wronskian formula

$$\mathrm{u}(x,t)=0 \tag{16}$$

where

$$\mathrm{W}=\mathrm{W}(\cosh(k_1\,(x-4\,k_1^{\;2}\,t)),\sinh(k_2\,(x-4\,k_2^{\;2}\,t))\,..\,\psi_n) \tag{17}$$

is the Wronskian determinant composed of

$$\psi_i(\xi_i) \tag{18}$$

and
$$\psi_i(\xi_i) \tag{19}$$
$$\xi_i \tag{20}$$
means
$$\xi_i = k_i\,(x - 4\,k_i^{2}\,t) \tag{21}$$
for
$$E_i < 0 \tag{22}$$
and
$$\xi_i = k_i\,(x + 4\,k_i^{2}\,t) \tag{23}$$
$$0 < E_i \tag{24}$$

Maple11 gives us:

$$Soliton := \textbf{proc}\,(w)\ \textbf{local}\ L;\ L := \ln(w);\ \mathrm{RETURN}(\mathrm{simplify}(2 \times \mathrm{diff}(L, x\ \$\ 2)))\ \textbf{end proc}$$

$$\mathrm{W} = \psi(\xi) \tag{25}$$

where $\psi(\xi) = \cosh(\xi)$ corresponds to a negative energy E of associated Schrodinger equation.

$$\xi := k\,(x - 4\,k^2\,t) \tag{26}$$
$$\psi := \cosh(k\,(x - 4\,k^2\,t)) \tag{27}$$

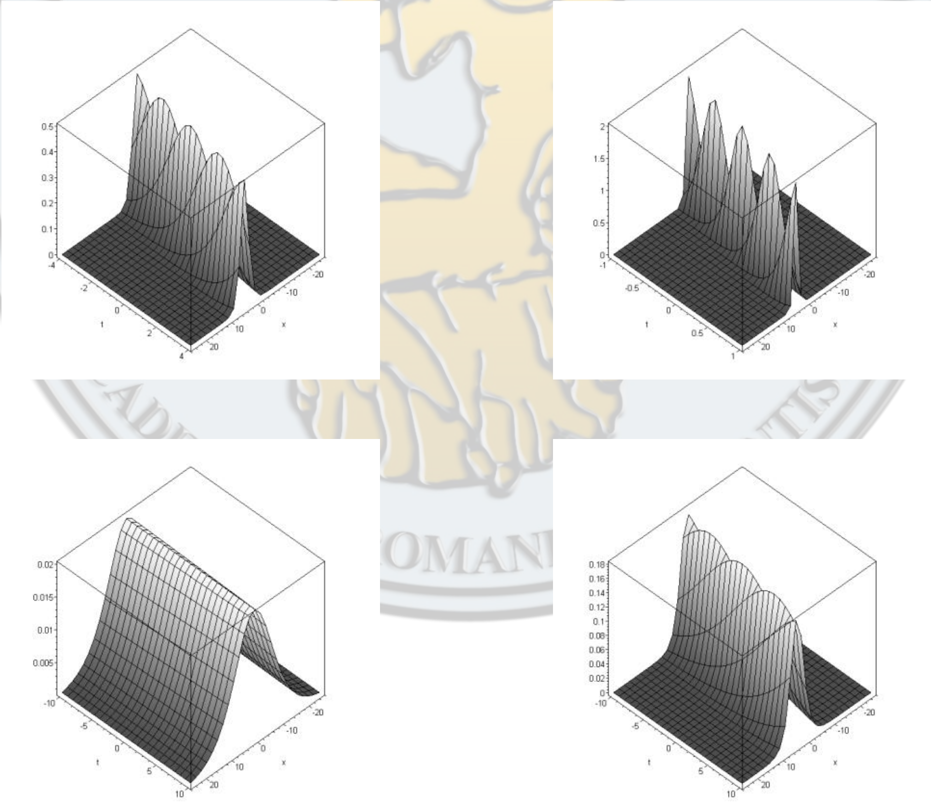

Fig. 3. The 3D visualization of *k* dependence of the equation (11)

The Wronksi matrix is $$MW1 := [\cosh(k\,(x - 4\,k^2\,t))] \quad (28)$$

and its determinant reads: $$\mathrm{W1} := \cosh(k\,(x - 4\,k^2\,t)) \,. \quad (29)$$

5. One Soliton Propagation. Numerical Results

Maple11 helps us visualize via 3D plotting the *k* dependence of the equation (11); for:

a) $k = 0.5$, $t \in [-4, 4]$ and $x \in [-25, 25]$;

```
> plot3d(2*.25/(cosh(.5*(x-4*.25*t)))^2, x = -25..25, t =-4..4);
```

b) $k = 1$, $t \in [-1, 1]$ and $x \in [-25, 25]$;

```
> plot3d(2/(cosh((x-4*t)))^2, x = -25..25, t = -1..1 );
```

c) $k = 0.1$, $t \in [-10, 10]$ and $x \in [-25, 25]$;

```
> plot3d(2*.01/(cosh(.1*(x-.04*t)))^2, x=-25..25, t=-10..10);
```

d) $k = 0.3$, $t \in [-10, 10]$ and $x \in [-25, 25]$;

```
> plot3d(2*.09/(cosh(.3*(x+4*.09*t)))^2, x=-25..25, t=-10..10);
```

we got the representations seen in figure 3 (a−d) and, obviously, we can study the pattern for whatever values of *k*.

The program Maple11 helps us plotting solutions (we can use animation or individual frames), for different values of the wave number *k*, for different moments *t* and for an arbitrary interval for *x*; we can study the dependence of speed and amplitude onto *k*.

Let $k = 0.6$, $t \in \{-6, -2, 2, 6\}$ and $x \in [-25, 25]$; Maple11 gives (see fig. 4):

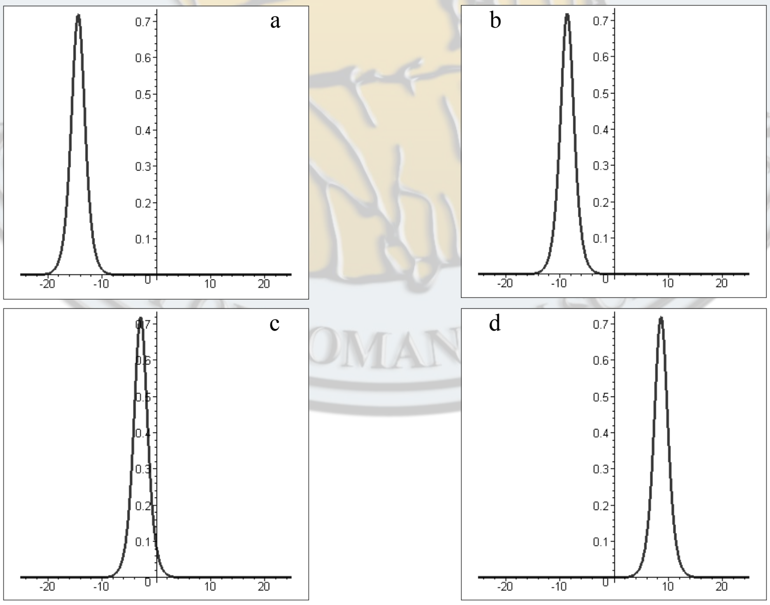

Fig. 4. Soliton propagation for $k = 0.6$ and $t \in \{-6, -2, 2, 6\}$: a, b, c, d respectively.

Now, for k = 0.2, $t \in \{-6, -2, 2, 6\}$ and $x \in [-25, 25]$; Maple11 gives (see fig. 5):

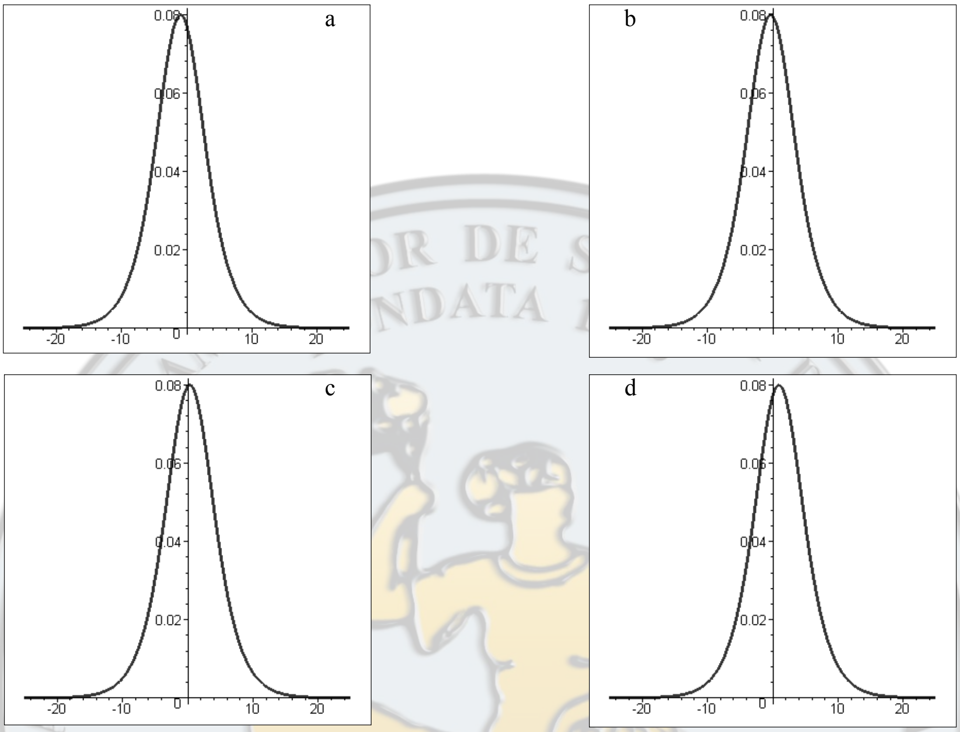

Fig. 5. Soliton propagation for $k = 0.2$ and $t \in \{-6, -2, 2, 6\}$: a, b, c, d respectively.

As we can see, both soliton's speed and amplitude are k dependent ([3], [12]).

6. Two Soliton Propagation. Numerical Results

For study the interaction between two solitons we need another solution of KdV equation ([1]).

We can obtain and verify this solution of the KdV equation by means of Maple11:

We consider in this case $W = W(\psi_1(\xi_1), \psi_2(\xi_2))$ where:

$$\psi_1(\xi_1) = \cosh(\xi_1)\,,\ \ \psi_2(\xi_2) = \sinh(\xi_2)$$

$$\xi_1 := k_1\,(x - 4\,k_1^{\,2}\,t)\,,\quad \xi_2 := k_2\,(x - 4\,k_2^{\,2}\,t)$$

$$\psi_1 := \cosh(k_1\,(x - 4\,k_1^{\,2}\,t))\,,\quad \psi_2 := \sinh(k_2\,(x - 4\,k_2^{\,2}\,t))$$

The Wronski matrix: $$MW2 := \begin{bmatrix} \cosh(k_1\,(x - 4\,k_1^{\,2}\,t)) & \sinh(k_2\,(x - 4\,k_2^{\,2}\,t)) \\ \sinh(k_1\,(x - 4\,k_1^{\,2}\,t))\,k_1 & \cosh(k_2\,(x - 4\,k_2^{\,2}\,t))\,k_2 \end{bmatrix}$$

has the determinant:
$$W2 := \cosh(k_1(-x+4k_1^2 t))\cosh(k_2(-x+4k_2^2 t))\,k_2 - \sinh(k_2(-x+4k_2^2 t))\sinh(k_1(-x+4k_1^2 t))\,k_1$$

and the corresponding two-soliton solution:

$$u2 := -2\,\big(k_2^2\cosh(k_1(-x+4k_1^2 t))^2 k_1^2 - k_2^2 k_1^2\cosh(k_2(-x+4k_2^2 t))^2 + k_2^2 k_1^2 + k_1^4\cosh(k_2(-x+4k_2^2 t))^2 - k_1^4 - \cosh(k_1(-x+4k_1^2 t))^2 k_2^4\big) \Big/ \big(-\cosh(k_1(-x+4k_1^2 t))\cosh(k_2(-x+4k_2^2 t))\,k_2 + \sinh(k_2(-x+4k_2^2 t))\sinh(k_1(-x+4k_1^2 t))\,k_1\big)^2$$

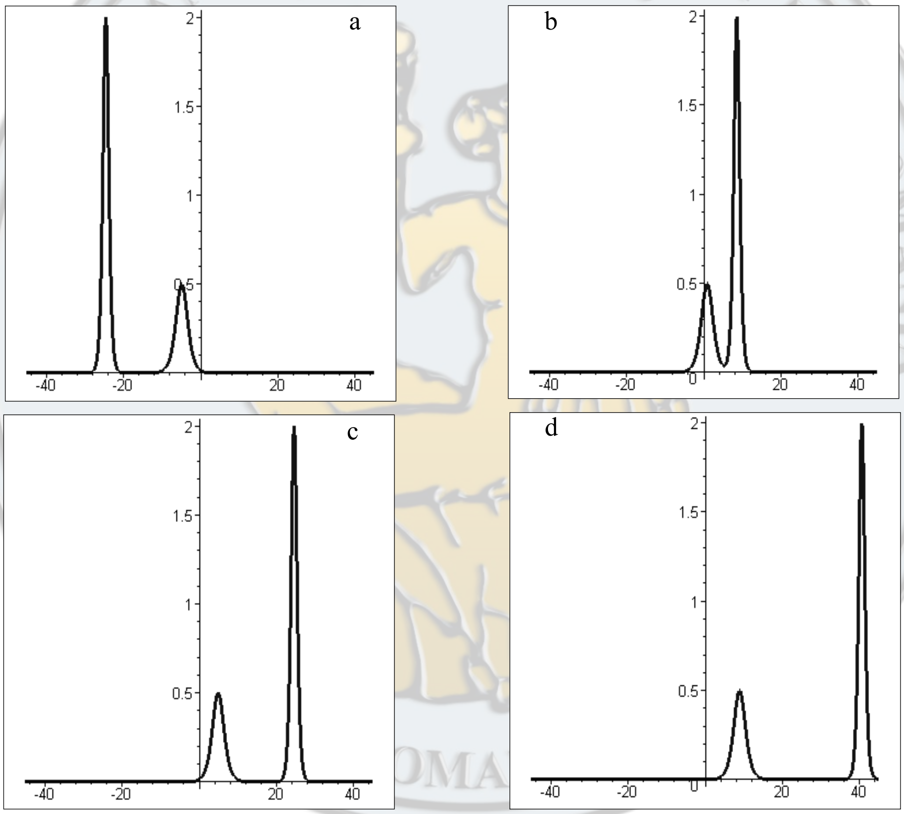

Fig. 6. The interaction of two solitons, for: $k_1 = 0.5$, $k_2 = 1$, $t \in \{-6, 2, 6, 10\}$

We check with Maple11 that it is a solution of KdV equation:

$$\left(\frac{\partial}{\partial t}u(x,t)\right) + 6\,u(x,t)\left(\frac{\partial}{\partial x}u(x,t)\right) + \left(\frac{\partial^3}{\partial x^3}u(x,t)\right) = 0.$$

The second solution is:

$$u2 := 2\,(-k_2^{\,2}\,k_1^{\,2}\cosh(k_1\,(x-4\,k_1^{\,2}\,t))^2 + k_2^{\,2}\,k_1^{\,2}\cosh(k_2\,(x-4\,k_2^{\,2}\,t))^2 - k_2^{\,2}\,k_1^{\,2}$$
$$- k_1^{\,4}\cosh(k_2\,(x-4\,k_2^{\,2}\,t))^2 + k_1^{\,4} + \cosh(k_1\,(x-4\,k_1^{\,2}\,t))^2\,k_2^{\,4}) \Big/ ($$
$$\cosh(k_1\,(x-4\,k_1^{\,2}\,t))\cosh(k_2\,(x-4\,k_2^{\,2}\,t))\,k_2$$
$$- \sinh(k_2\,(x-4\,k_2^{\,2}\,t))\sinh(k_1\,(x-4\,k_1^{\,2}\,t))\,k_1)^2 \qquad (30)$$

With these solutions, Maple11 helps us visualize the interaction between the two solitons (Fig. 6).

We can see both solitons regaining their shape after *collision* (i.e. after they pass one through another). Obviously, is better to use the animation feature of the Maple11 program, modify the parameters and observe the changes.

7. Three Soliton Propagation. Numerical Results

For the tri-solitonic solution one may obtain and verify (by means of Maple11) the following solution:

$$\operatorname{sech}^2\left(\frac{\sqrt{a}\cdot(x-2\cdot t\cdot a)}{\sqrt{2}}\right)\cdot a - \left(2\cdot(b-c)\right.$$
$$\cdot\left(\frac{2\cdot(c-a)\cdot\left(\operatorname{sech}^2\left(\frac{\sqrt{c}\cdot(x-2\cdot t\cdot c)}{\sqrt{2}}\right)\cdot c-\operatorname{sech}^2\left(\frac{\sqrt{a}\cdot(x-2\cdot t\cdot a)}{\sqrt{2}}\right)\cdot a\right)}{\left(\sqrt{2\cdot c}\cdot\tanh\left(\frac{\sqrt{c}\,(x-2\cdot t\cdot c)}{\sqrt{2}}\right)-\sqrt{2\cdot a}\cdot\tanh\left(\frac{\sqrt{a}\,(x-2\cdot t\cdot a)}{\sqrt{2}}\right)\right)^2}\right.$$
$$\left.\left.-\frac{2\cdot(a-b)\cdot\left(\operatorname{csch}^2\left(\frac{\sqrt{b}\cdot(x-2\cdot t\cdot b)}{\sqrt{2}}\right)\cdot b+\operatorname{sech}^2\left(\frac{\sqrt{a}\cdot(x-2\cdot t\cdot a)}{\sqrt{2}}\right)\cdot a\right)}{\left(\sqrt{2\cdot a}\cdot\tanh\left(\frac{\sqrt{a}\,(x-2\cdot t\cdot a)}{\sqrt{2}}\right)-\sqrt{2\cdot b}\cdot\coth\left(\frac{\sqrt{b}\,(x-2\cdot t\cdot b)}{\sqrt{2}}\right)\right)^2}\right)\right)\Bigg/$$
$$\left(\frac{2\cdot(a-b)}{\sqrt{2\cdot a}\cdot\tanh\left(\frac{\sqrt{a}\,(x-2\cdot t\cdot a)}{\sqrt{2}}\right)-\sqrt{2\cdot b}\cdot\coth\left(\frac{\sqrt{b}\,(x-2\cdot t\cdot b)}{\sqrt{2}}\right)}\right.$$
$$\left.-\frac{2\cdot(c-a)}{\sqrt{2\cdot c}\cdot\tanh\left(\frac{\sqrt{c}\,(x-2\cdot t\cdot c)}{\sqrt{2}}\right)-\sqrt{2\cdot a}\cdot\tanh\left(\frac{\sqrt{a}\,(x-2\cdot t\cdot a)}{\sqrt{2}}\right)}\right)^2$$

where a, b, c are real, positive constants.

Maple11 permits us to make some useful plots (see fig. 7) – specially animated plots - choosing various values for the parameters *a*, *b* and *c*.

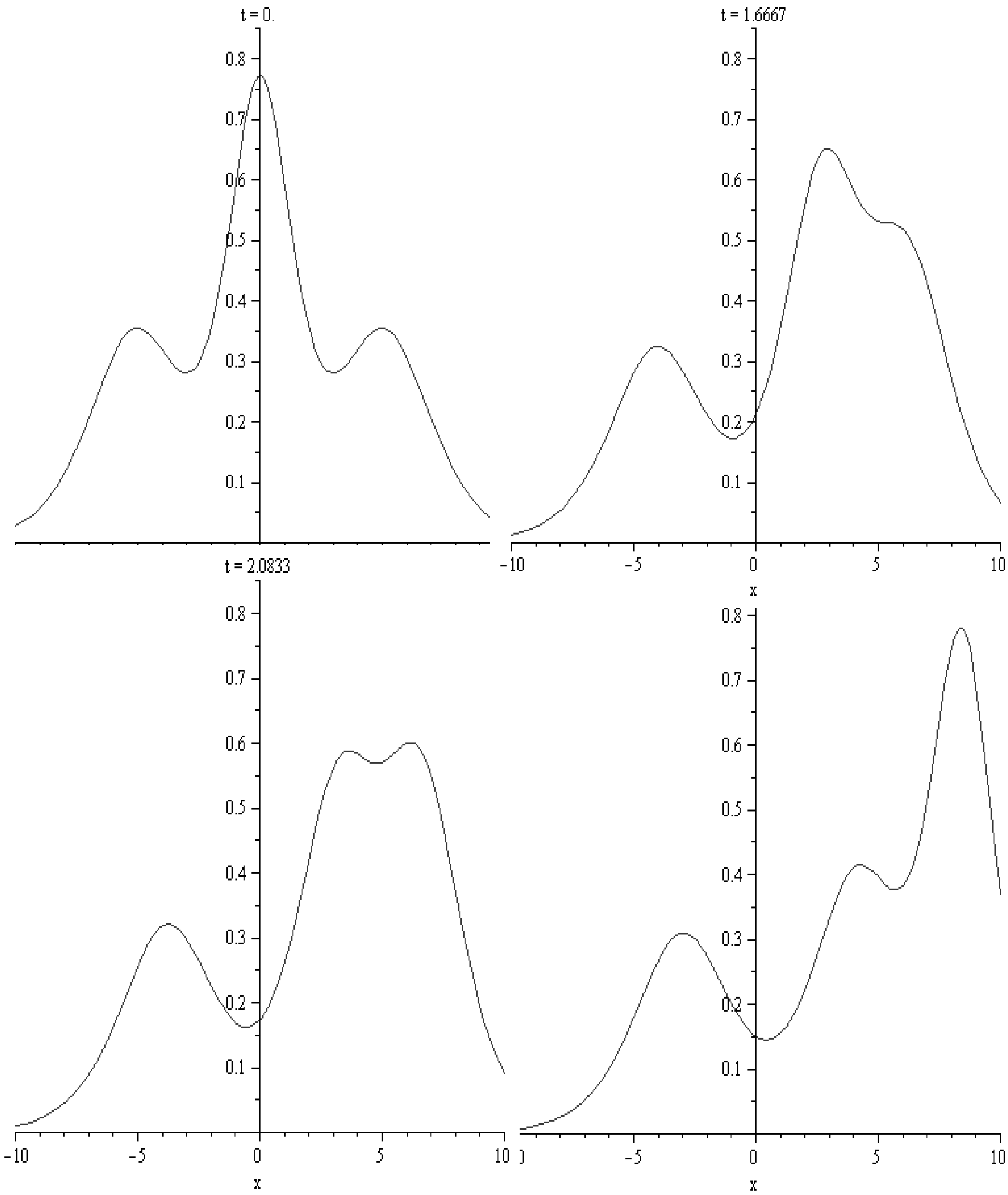

Fig. 7. Animated plots for the three solitons interaction.

8. Conclusions

Those computer experiments are very suitable for proofing to students in optical engineering and designers of optical circuits the main features of soliton propagation in optical fibers, as follows, reproducing those from [1].

The fundamental soliton appears as solutions of NLS equation.

Solitonic solutions may be obtained solving KdV equation. Solving equation by means of Maple11 program is useful because is quick, but the solution must be verified *by hand*.

The existence of solitons is due to the balance between the dispersion and the nonlinearity of the medium.

Both soliton's speed and amplitude are *k* dependent; this dependence may be easily observed by modifying *k* in equation (14) and (15).

For designers working in digital data transmission, the simulation of the solitons propagation along optical fibers is useful and may be performed successfully with Maple11.

The animated plots and the 3D plots also are some useful features of this computer program; especially: the collision of two or three solitons and the *k* dependence of a soliton propagation can be studied by this means respectively.

Solitons scatter elastically; after the collision, solitons regain their original shape and velocity ([8], [9], [15]).

The only remaining effect of the scattering is a phase shift (i.e. a change in the position they would have reached without interaction).

For solitons keep invariant shape and size is in accord with the conservation laws, one can infer that a profound link exists between integrable models and the theory of solitons.

R E F E R E N C E S

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