STUDY OF THE INFLATION STAGES OF SOME GROWTH PROCESSES

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Rezumat. *Pentru a înţelege rolul informaţiei genetice pentru procesele de creştere ale diferitelor vieţuitoare, identificarea etapelor care lansează aceste creşteri este extrem de importantă. Desigur, pentru a evita orice posibilă interpretare eronată a informaţiilor căpătate, este necesară o examinare minuţioasă şi sistematică a rezultatelor obţinute. Lucrarea abordează această temă, evidenţiind de asemenea unele asemănări cu unele etape ale evoluţiei Universului. Datorită generalităţii lor, acest studiu foloseşte în principal modelele de similitudine ale proceselor de creştere/adaptare.*

Abstract. *In order to understand the role of the genetic information for the growth processes of the living beings, the identification of the stages that launch this growth is very important. Of course, a thorough and systematical examination of the obtained results is necessary in order to avoid any possible wrong interpretations of the received information. This work deals with this topic, pointing out also some similarities with some stages of the Universe evolution. Due to their generality, this study uses basically the similitude models of the growth/adaptation processes.*

Key words: similitude models, growth processes, Universe evolution, human being growth, inflation stages

1. Introduction

The growth processes have either: (i) a continuous character, or they present: (ii) frequent second type (fractal) discontinuities. Taking into account: a) the existing rather detailed theoretical examinations and descriptions of the fractal growth processes [1], [2], b) that the usual descriptions of the growth processes refer mainly to the growth stagnation and extinction [3]-[5], not to its launching, c) the important advantages of the similitude models of physical systems, we will examine mainly the main features of the similitude models of the launching (inflation) stages of the continuous growth processes. In this aim, the main stages of the physical systems modeling, as well as of the study of the compatibility of theoretical models relative to the existing experimental data, are studied.

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2. Basic stages of the obtained information processing

a) Definitions of the true information amount and of the error risk

As it is well-known, unlike Mathematics whose basic element is a numerical quantity (the solution of an equation, the length of a geometrical figure side, etc), in Nature Sciences – the basic element is given by a distribution (see Figure 1). Due to different errors, the result of an experimental study corresponds to a certain distribution, that does not coincide with the true one (fig. 2).

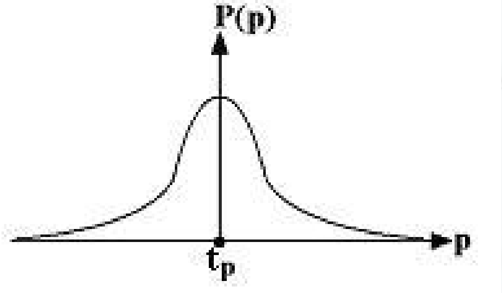

Fig. 1. The probability distribution of the individual values of a certain physical parameter

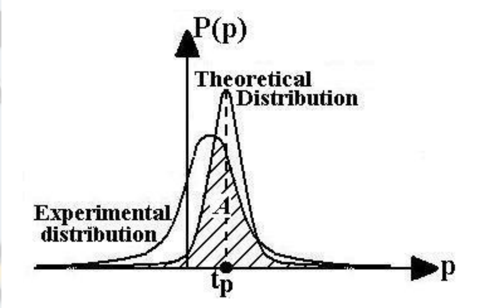

Fig. 2. Comparison of the theoretical distribution and of the (experimentally) obtained information (distribution)

If both distributions are normalized : $\int_{-\infty}^{\infty} P(p)\cdot dp = 1$, then : (i) the shaded (common) area A corresponds to the error risk at the rejection of the compatibility of the theoretical distribution with the experimental one, b) the quantity $\Im = 2A - 1$ represents the obtained true information: if $\Im > 0$ then the experimental study led to some useful information on the theoretical distribution, while for $\Im < 0$ the obtained distribution corresponds to a misinformation [6].

b) Classical stages of the physical processes modeling

The modeling (starting from the experimental results) of the physical processes was examined in detail in the specialty literature [7], [8]. The analysis of the main types of present physical models points out the presence of the following basic stages:

a) the identification of the uniqueness parameters of the studied state (process),
b) the identification of the characteristic similitude criteria [8],
c) the finding of the set of irreducible similitude,
d) the expression of all relations of interest for applications or scientific or scientific descriptions in terms of some similitude criteria, exclusively,
e) the test (check out) of the theoretical and of the experimental similitude models,
f) the test of the compatibility of the similitude models relative to the experimental results.

c) Study of the compatibility of theoretical models relative to the experimental data

In order to test the compatibility of the similitude models with the experimental results, one starts from the finding that - because any physical relations or laws represent only some approximations of the empirical results, due to the increase of the accuracy of the accomplished measurements – all these relations will be found as inaccurate, the basic decision in the statistical studies of the experimental studies being so the *rejection of the compatibility of some relations (and theoretical models) with the studied experimental results.*

As to any statistical hypothesis, it is associated a certain error risk also to the hypothesis of compatibility rejection, that should be always known, but that is seldom studied! In order to advance in this direction, we will start from the finding that – as it is known: a) to each set of experimental results concerning N different parameters corresponding to the same state of the studied system (let x_{1mp} ,...x_{Nmp} – be the most probable values of these parameters) it is associated a confidence domain, that – in the frequent case of a normal distribution – has the shape of a N-dimensional ellipsoid: $\varepsilon^T \overline{\overline{\Gamma}}^{-1} \varepsilon = f_N(L_i)$, (1)

where is the "column" vector of the deviations (errors): $\varepsilon_j = x_j - x_{j,mp}$ ($x_{j,mp}$ being the most probable value of the parameter X_j), $\bar{\varepsilon}^T$ is the transposed of the ("row") vector $\bar{\varepsilon}$, $\overline{\overline{\Gamma}} = \left|\Gamma_{ij} = \left\langle \left(x_i - x_{i,mp}\right)\left(x_j - x_{j,mp}\right)\right\rangle\right|$ is *the co-variances matrix*, and $f_N(L_i)$ is a certain function on the confidence level L_i corresponding to the considered confidence domain, b) in the frequent case of the study of a pair of physical parameters (X and Y), the confidence domain corresponding to the normal 2D distribution will correspond to the internal part of the ellipse:

$$\varsigma^2(x_k) + \varsigma^2(y_k) - 2r_k \varsigma(x_k)\cdot\varsigma(y_k) = f_2(L_k) = -2\left(1 - r_k^2\right)\cdot \ln\left(1 - L_k\right), \quad (2)$$

where: (i) $s(x_k)$ and $s(y_k)$ are the square mean errors corresponding to the values of the parameters X and Y for the state k, (ii) $\varsigma(x_k) = \dfrac{x_k - x_{k,cmp}}{s(x_k)}$, (3)

are the corresponding reduced errors, and: (iii) r_k is the correlation coefficient of the values of these parameters for the studied state:

$$r_k = \frac{\Gamma(x_k, y_k)}{s(x_k)\cdot s(y_k)} = \frac{<(x_k - x_{k,cmp})(y_k - y_{k,cmp})>}{s(x_k)\cdot s(y_k)} \ . \quad (4)$$

Usually, the correlation coefficient r_k is considered as the main criterion for the appreciation of the compatibility of certain theoretical relations $y=f(x)$ relative to some experimental data. In fact, this coefficient "measures" only the proximity degree of the confidence domains centers relative to the regression line (curve); e.g., despite that $|r_a|>|r_b|$, the ensemble of experimental values from fig. 3a is not compatible with relation *y=f(x)*, while the experimental data set from fig. 3b is compatible with this relation, because the corresponding confidence domains are crossed by the regression curve (line) *y = f(x).* Obviously, the solution of such problems, very important for the experimental data processing can be accomplished only by means of computers. Particularly, some too less values (e.g., less than 0.01) of the error risk q_k=1-L_{ik} (obtained from relations (2) and (4) for $x_k=x_{tk}$, $y_k=y_{tk}$, where x_{tk}, y_{tk} are the coordinates of the tangency point of the confidence ellipse to the regression curve (line) *y=f(x)* (see the dotted ellipse from Fig. 3b), can justify the incompatibility hypothesis of the studied relation *y=f(x)* relative to the considered experimental data sets [9].

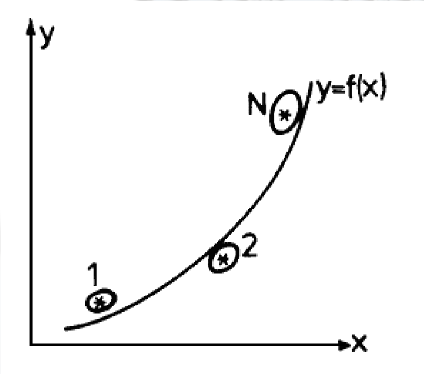

Fig. 3a. The incompatibility relative to the experimental data can intervene sometimes even for rather high correlation coefficients!

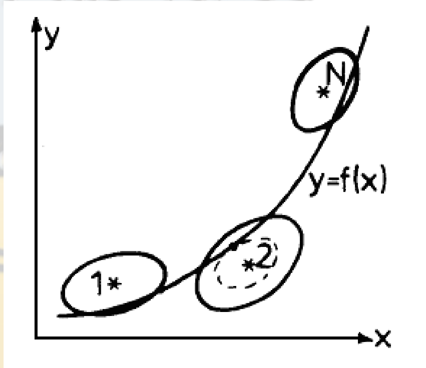

Fig. 3b. The compatibility with the experimental data is sometimes possible even for a rather small value of the correlation coefficient

As the error risk: $q_k = 1 - L_{ik} = \exp\left[-\frac{(x_{tk} - \langle x_{ki} \rangle)^2}{s^2(x_k)} - \frac{(y_{tk} - \langle y_{ki} \rangle)^2}{s^2(y_k)} \right]$ (5)

at the rejection of the compatibility of the experimental results corresponding to the state *k* relative to the studied theoretical relation *y = f(x)* is less, or it is larger than a certain threshold (usually between 0.01 and 0.2), the studied compatibility is rejected, or it is accepted, resp. [9] ($\langle x_{ki} \rangle, \langle y_{ki} \rangle$ are the statistical averages of the individual values of the parameters *X, Y* corresponding to the state *k*).

3. Classical similitude models of the growth processes

Starting from the differential equation of the growth (accommodation) of an arbitrary physical parameter *Y(t)*: $\frac{dY}{dt} = \pi(t) \cdot Y(t)$, where: $[\pi(t)] = \frac{1}{T}$, (6)

i.e. $\pi(t)$ is the time density of the growth (accommodation) probability and introducing the similitude criteria (functions):

$$\tau = \pi(0)\cdot t\,,\quad y(t) = \frac{Y(t)}{Y(0)}\ \text{, and: }\ p(t) = \frac{\pi(t)}{\pi(0)}\ , \tag{7}$$

one obtains *the similitude growth equation*: $\frac{dy}{d\tau} = p(\tau)\cdot y(\tau)$, i.e.: $p(\tau) = \frac{dz}{d\tau}$, (8)

by means of the similitude variable: $z = \ln y$. (9)

Assuming that z is a function solely of $p(\tau)$, one obtains:

$$\dot{p} = \frac{\dot{z}}{dz/dp} = \frac{p}{\sum_{n=0}^{\infty}\varepsilon_n p^n} = \sum_{n=1}^{\infty}\alpha_n\cdot p^n(z)\ . \tag{10}$$

The growth processes can be classified according to the degree N of the algebraic polynomial $\dot{p}(p)$ in the following universality classes [5e]: a) U0 (corresponding to a constant value of the probability density $p(\tau)$), b) U1, for a linear $\dot{p} = f(p)$ dependence, … c) U3, for a $\dot{p} = f(p)$ dependence expressed by a 3rd degree polynomial, etc.

b) The particular case of the auto-catalytic growth processes (U0 model)

Many results concerning the basic features of the growth processes in Physics [10], Biology [11] and even Cosmology [12], point out the outstanding importance of the auto-catalytic (exponential) growth type. *The auto-catalytic growth equation* can be obtained from equation (6), for: $\pi(\tau) = const. = \frac{1}{\tau_{ac}}$, where τ_{ac} is the time constant of the auto-catalytic growth: $Y(\tau) = \exp\left(\frac{\tau}{\tau_{ac}}\right)$. (11)

c) The particular case 2: the U1 (Gompertz's) model [3]

The Gompertz's model was proposed in 1825, in strong connection with the study of some tables concerning the mortality. Starting from relation (10), with null coefficients, excepting $\alpha_1 = -1$, one obtains: $dp/d\tau = -p$. (12)

Integrating the equation (8) with the probability time density (12), one obtains:

$$\ln y = -\exp(-\tau) + C\,,\ \ \text{hence:}\ \ y = \exp[C - \exp(-\tau)]\ ; \tag{13}$$

from the condition: $y(0) = 1$, one finds the similitude expression of the growth equation, according to Gompertz's model: $y = \exp[1 - \exp(-\tau)]$. (14)

The Gompertz's model is valid for some tumors growth processes [13], [14], as well as for the descriptions of the population dynamics [15], etc.

d) *The 3rd particular case: the U2 (West's) model* [4], [5a]

The similitude expression of the U2 (West's) growths [4], [5a]:

$$y = \left[1 + b - b \cdot \exp(-\tau)\right]^{1/b} \,, \tag{15}$$

generalizes the equation (13), because it leads to the similitude expression of the Gompertz's growth for $b \to 0$. Obviously, the growth rate is given in this case by the nonlinear expression:

$$\frac{dy}{d\tau} = \frac{1+b}{b} \cdot y^{1-b} - \frac{1}{b} \cdot y \,. \tag{16}$$

Figure 4 presents the similitude plots of the growths of U2 (West's) type for different values of the parameter *b*, the value *b* = 0 corresponding to the Gompertzian growths, while the value $b = +0.25$ corresponds to the original West's model [4].

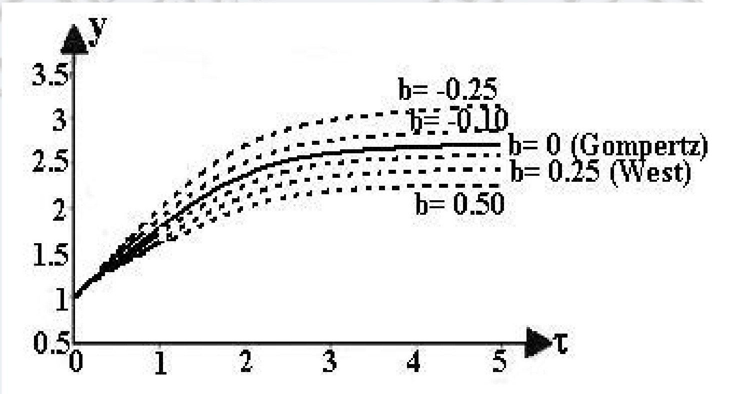

Fig. 4. The nondimensional (similitude) plots of the growths corresponding to the U2 (West's) model [4], [5a]

We consider as useful to point out that the U2 (West's) model:

(i) describes the growth processes of the living beings, in the range protozoa – plants – mammals, by means of the similitude equation:

$$m(\tau) = \left[M^{b} - b \cdot \exp(-\tau)\right]^{1/b} \,, \text{ where: } M = \lim_{\tau \to \infty} m(\tau) = (1+b)^{1/b} \,; \tag{17}$$

defining *the development remainder (rest)* by means of the expression:

$$z = 1 - \left(\frac{m}{M}\right)^{b} \,, \tag{18}$$

one finds that: $z = \exp(-\theta)$, where: $\theta = \tau + \ln b - b \cdot \ln M$ (19)

is the so-called *biological time*,

(ii) can be used for the tumors growth description, with values of the parameter *p* depending on the fractal nature of the biological channels (e.g. in angio-genesis) [5].

d) *The 4th particular case: the U3 (Delsanto's) model* [5b], [5f]
Starting from the generalization of the expression (13) of the similitude rate of the growth of the probability density: $\dot{p} = -p\left(1 + b \cdot p + c \cdot p^2\right)$, (20)

one obtains [see relations (7) and (12)]: $-\ln y + K = -\int \frac{dp}{p \cdot} = \int \frac{dp}{1 + b \cdot p + c \cdot p^2}$. (21)

One finds that – the U3 (Delsanto's) model involves an additional parameter (c) relative to the previous U2 (West's) model – the model expressed by the equation (21) presents important advantages for the detailed description of some particular features of the growth processes [5f].

e) *Some typical graphic representations of the basic growth processes*
Starting the similitude variable z defined by relation (9), we propose the use of representations in the "phases space": $z = ln$y and $\dot{z} = dz/dt$, because then:

(i) the equation of the auto-catalytic (exponential) growth corresponds to a horizontal straight-line segment: $\dot{z} = s\ (constant)$, and: $z = ln$y $= m + s.t$,

(ii) the equation of the U1 (Gompertz) growth is [16]:

$$z = \ln y = \ln y_o - \frac{a_o}{\alpha_1} + \frac{1}{\alpha_1} \cdot \dot{z} \ \ (\alpha_1 < 0),$$

equivalent to: $\dot{z} = a_o + \alpha_1 (z - z_o)$, where: $a_o = \dot{z}_o$ and: $z_o = \ln y_o$, (22)

(iii) the equation of the U2 (West's) growth is [16]:

$$\dot{z} \equiv a = \left[\left(\frac{1}{a_o} + \frac{\gamma}{\beta}\right) \cdot e^{-\beta t} - \frac{\gamma}{\beta}\right]^{-1} \quad \text{and:} \quad e^{-\gamma z} = y^{-\gamma} = \frac{1}{a_o} + \frac{\gamma}{\beta}\left(1 - e^{-\beta t}\right) = \frac{e^{\beta t}}{\dot{z}},$$

leading to: $$\dot{z} = \left(1 + \frac{\beta}{\gamma \cdot a_o}\right) \cdot e^{\gamma z} - \frac{\beta}{\gamma}. \quad (23)$$

One finds so that the shape (in the phases space $\dot{z}, z$) of the: (i) *auto-catalytic growth* is that of a "horizontal" straight-line segment, (ii) *Gompertz growth* is that of a descending (of negative slope) straight-line segment, (iii) *West- Delsanto's growth* is that of a relaxing (because $\gamma < 0$, see e.g. [16]) exponential.
Taking into account that the studied experimental data concerning the human growth (in its embryo and child phases, especially [17] - [19], see below) indicate the existence of some growth stages with positive slopes of the $\dot{z} = f(z)$ plots, we will name these stages as **phases of inflation growth** (figure 5).

(iv) all growth equations can be obtained from the general expressions:

$$\frac{dz}{\dot{z}(t)} = dt \ , \quad \text{i.e.:} \ t = \int \frac{dz}{\dot{z}(t)} \ .$$

As a particular example, in the case of the U_1 (Gompertz) evolution, one finds:

$$t = \int \frac{dz}{\alpha_1 (z - z_o) + a_o} = \frac{1}{\alpha_1} \ln\left(z - z_o + a_o / \alpha_1\right) - \frac{1}{\alpha_1} \ln \frac{a_o}{\alpha_1},$$

hence: $$\frac{a_o}{\alpha_1} \cdot e^{\alpha_1 t = z - z_o + \frac{a_o}{\alpha_1}} \quad \rightarrow \quad z - z_o = \frac{a_o}{\alpha_1}\left(e^{\alpha_1 t} - 1\right),$$

i.e. the Gompertz's growth equation.

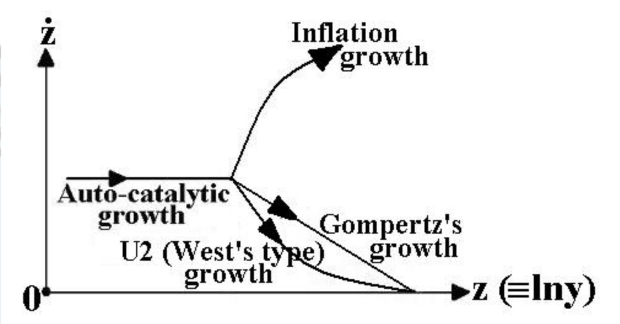

Fig. 5. Plots of the main types of growth processes in the diagram $(\dot{z}, z \equiv \ln y)$

4. Analysis of some existing experimental results referring to different growth/adaptation processes

a) *The Universe growth (expansion)*

The existing experimental data referring to the Universe evolution (growth, expansion) were synthesized by some theoretical studies, the Guth-Linde model (see [20], [21] and figure 6) of the inflationary Universe, following its Big Bang appearance, being presently accepted even by some academic textbooks [22].

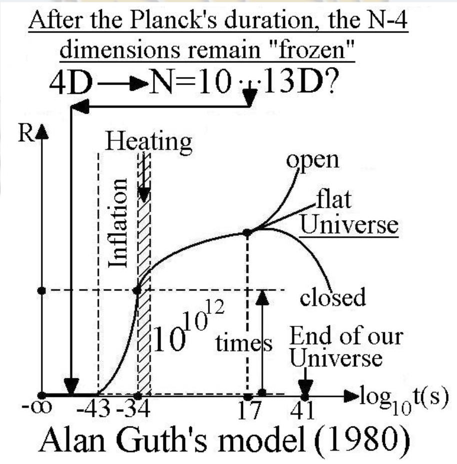

Fig. 6. Qualitative plot of the Universe evolution according to Linde [21d]

We believe that it is necessary to underline the following basic findings: (i) the presence of an initial “germination” phase (stage), (ii) the extraordinary quick rise of the Universe size ($10^{10^{12}}$ times [22d], required by the present theoretical descriptions) during the “inflation” stage, (iii) the use of a double-logarithmic scale for the representation of the Universe growth from fig. 4.

We consider as useful to underline from beginning that the “inflation”[3] phase of our Universe generation is considered as absolutely necessary to explain our Universe building both by the lay (laic) specialists (by the authors of the American academic textbooks from our century [22], particularly), as well as by the theist cosmologists [23].

The “inflation” phase is now accepted by all specialists because our Universe has to be:

(i) “cleared” of super-heavy particles (of the magnetic mono-poles, especially),

(ii) described (with some local exceptions, seldom met) by the Euclidean geometry,

(iii) rather homogeneous, even in conditions when some information (e.g. relative to its local temperatures) cannot be transmitted in the limits of the special relativity (the “horizon” problem, see above), etc.

As it was found [24], the classical (standard) Big Bang model ensures a multiplication factor (of the Universe) of approx. 10^{30} times, that is absolutely insufficient for our Universe description. For the obtainment of values corresponding to the observed distances (10^{26} *m*) it is necessary a multiplication factor considerably larger (of about 10^{60} times), but … even this magnitude order cannot explain the manner of the magnetic mono-poles disappearance from our Universe!

In fact, the Guth – Linde’s model of inflation [20], [21], assumes an incredibly large multiplication factor (of about $10^{10^{12}} = 10^{1000,000,000,000}$!), but that allows

[3] The Big Bang models of the “inflation” type were introduced approximately concomitantly by the American cosmologists [20], and those of the Russian school [21], resp. [it seems that there existed some non-rigorous models of this kind even since 1972 (D. A. Kirznits and A. Linde), the first such model considered as realistic (but heavy) of the inflation being due to A. A. Starobinski (1979)].

to explain why – in the conditions of the additional action of some friction-like forces – the super-heavy particles (of the type of magnetic mono-poles) were "removed" at practically infinite distances from our Universe, while the considerably lighter particles (as the protons) remained at the "end", in the limits of the Universe known by us.

This "inflation" phase presents the following basic features:

(i) it is an somewhat exponential growth phase of the Universe, essential element in order to explain its fractal properties,

(ii) it eliminates (as it was shown above) the super-heavy particles (the magnetic mono-poles) from our Universe,

(iii) it ensures (with local exceptions, relatively seldom met) a high homogeneity of our Universe, that explains its "flatness" [the validity – with the exceptions of local substance agglomerations (stars, black holes, etc) – of its Euclidean geometry],

(iv) it assumes "the operation" of some considerably larger velocities than the light speed in vacuum, hypothesis that solves the "horizon" problem, by the transmission at very large distances of information (about the temperature, particularly) even from this phase.

Of course, the above findings show that the laws of the contemporary Physics did not act in the inflation phase of the Universe.

If the inflation stage could correspond to an auto-catalytic growth, its equation would be:

$$\ln\left(R(t)/R_P\right) = \frac{1}{\tau}\left(t - t_P\right) , \tag{24}$$

where $R_P \left(\cong 1.6 \cdot 10^{-35} m\right)$ and $t_P \left(\cong 0.533 \cdot 10^{-43} s\right)$ are the Planck's radius and time, resp., while τ is the corresponding characteristic time. At the end of the inflation stage, we would have:

$$\ln\left(R_{ie}/R_P\right) = \ln\left(10^{10^{12}}\right) \cong 2.3 \times 10^{12} = \frac{1}{\tau}\left(t_{ie} - t_P\right) \cong 10^{-35}/\tau \text{, hence: } \tau \cong 4.35 \times 10^{-48} s .$$

Because it is improbable that the characteristic time could be less than the Planck's time $t_P \left(\cong 0.533 \cdot 10^{-43} s\right)$, that has a somewhat character of time quantum, it results that the time dependence of the Universe size should be

stronger than the usual exponential one, corresponding to the auto-catalytic growth. Assuming that: $\ln\left(R(t)/R_P\right)=\left[\left(t-t_P\right)/\tau\right]^m$, with $m > 1$, (25)
and that: $\tau = t_P$, one obtains: $\ln\left[\ln\left(R_{ie}/R_P\right)\right]=m\cdot\ln\left[\left(t_{ie}-t_P\right)/\tau\right]$ and:

$$2.3025\left[\lg 2.3025+12\right]\cong m\cdot\ln\left[10^{-35}/0.533\times10^{-43}\right]\cong m\cdot 2.3025\left[\lg\left(\frac{1}{0.533}\right)+8\right],$$

hence: $m\cong\dfrac{12+\lg 2.3025}{8+\lg 1.876}\cong 1.494$, that seems to be plausible.

The assumed time dependence (25) during the inflation stage can be written as:

$$z\equiv\ln\left(R(t)/R_P\right)=\left[\frac{1}{\tau}\left(t-t_P\right)\right]^m, \quad \text{hence: } \dot{z}=\frac{m}{\tau}\left[\frac{1}{\tau}\left(t-t_P\right)\right]^{m-1}. \tag{26}$$

Because from the assumed evolution equation it results: $\dfrac{1}{\tau}\left(t-t_P\right)=z^{1/m}$, substituting in the previous relation one finds: $\dot{z}=\dfrac{m}{\tau}\cdot z^{(m-1)/m}=C\cdot z^n$, (27)

where: $n=\dfrac{m-1}{m}\in\left(0,1\right)$ and: $C=\dfrac{m}{\tau}>0$. (27')

One finds so that the $\dot{z}=f(z)$ plot of an inflation growth is that from figure 3. We can observe that a such plot corresponds also to the height $z(t)$ evolution of a rocket launched vertically upward, whose acceleration diminishes in time.

b) *Accommodation (adaptation) processes. The model of the auto-catalytic – stagnation processes coupling* [25]

As it results from figure 5, the growth models of the Gompertz's type do not involve usually some inflexion points of the growth (adaptation) characteristics, as it was found rather frequently both in Physics [10] and Biology[4] [11], and even (as it seems) in Cosmology [12]. A typical plot of the accommodation (adaptation) processes is that of the skeletal muscles contractions under the action of an increased concentration of the Ca^{2+} myoplasmic ions (see figure 7 and [25]). Assuming a uniform injection of the Ca^{2+} myoplasmic ions, figure 7 can be generalized for the coupling of any arbitrary auto-catalytic and stagnation stages

[4] We will mention also the finding of *Science et Vie*, no. 1079, p. 51 (August 2007) "Chez les bactéries, qui évoluent plus vite que les organismes complexes, les plus fréquentes sont ainsi ces mutations dites "neutres", c'est-à-dire sans effet notoire (avec une probabilité d'une sur 1000 par divisions, en moyenne, comme l'ont observé les biologistes comparant les génomes de plusieurs générations de bactéries); viennent ensuite les néfastes (une sur 10.000); puis les mortelles (une sur 100.000); et, **enfin, les bénéfiques (une sur 10.000.000)**". The evolution (if it produces) is due so to the extremely fast increase (exponential, by means of some auto-catalytic growth processes) of the favorable mutations.

intervening in the description of the accommodation (growth) processes of different parameters *p(t)*, as it is shown by figure 8.

For this reason, we studied also the possibilities of description of some growth (adaptation) phenomena, synthesized (for an arbitrary parameter *p*) by means of the plot presented in figure 8.

The accomplished study pointed out [24] the possibility of descriptions of some growth (accommodation) processes like those indicated by figures 6 and 7 by means of some (master) differential equations of the type:

$$\frac{dp}{p} + \rho \cdot \frac{dp}{p - p_\infty} + \frac{dp}{p_{ms}} = \frac{dt}{\tau_{ac}} \,. \tag{28}$$

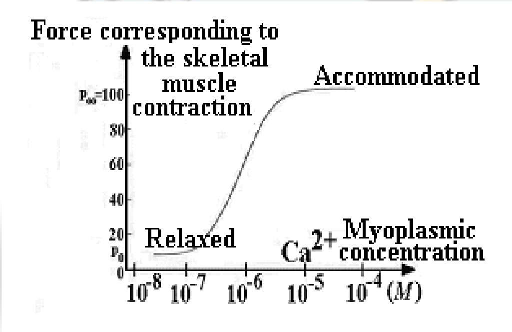

Fig. 7. The dependence on the myoplasmic Ca^{2+} concentration of the force corresponding to the skeletal muscle contraction [26]

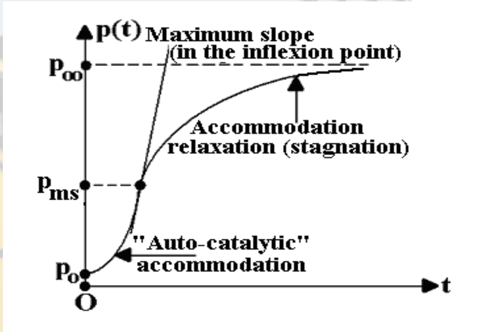

Fig. 8. Typical plot of the coupling of the auto-catalytic and stagnation growth processes

From the definition of the inflexion point of the growth (accommodation) plot:

$$\left(\frac{d^2 p}{dt^2}\right)_{\text{inf}} = 0 \text{ , one obtains: } p_{\text{inf}} = \frac{p_\infty}{1 + \sqrt{\rho}} \,, \tag{29}$$

and:

$$\dot{p}_{\max} = \dot{p}_{\text{inf}} = \frac{\rho}{\tau_{rel}} \left[\frac{\left(1 + \sqrt{\rho}\right)^2}{p_\infty} + \frac{1}{p_{ms}} \right] \text{, respectively.} \tag{30}$$

For the alnico alloy, we obtained [25]:

$$\tau_{rel} \approx 52 \text{ min and: } \frac{\dot{p}_{\text{inf}}}{p_\infty} \approx 0.1 \text{ min}^{-1}.$$

In some limited conditions, the predictions of this model and those of the U2 (West's type) model coincide (problem 10.4.1 [27]).

c) *Basic stages of the human being growth*

The basic stages of the human being growth are those of [16] – [18]: (i) embryo, (ii) fetal growth, (iii) baby, (iv) child, and (v) teenager growth. Some existing experimental data concerning these growth stages will be analyzed below.

(i) Embryo growth

The average values (indicated by references [16a-c]) of the human embryo length L (mm), were used to calculate the values of the similitude parameter $z_L = \ln\{L\}_{0.1\,mm}$ and those of its growth rate: $\dot{z}_L(t) = \dfrac{L(t+\Delta t) - L(t-\Delta t)}{2L_m \Delta t}$. The obtained results were synthesized by figure 9.

One finds that the similitude growth rate $\dot{z}$ and its opposite characteristic growth time τ_L present: a) a rather slow (or even null) growth rate during the "germination" phase (up to approximately 10 days), b) an absolute (for the entire human being growth) maximum at approximately 17 days (from fecundation), c) frequent oscillations (probably due to the empiric data rounding) around some average values, in the interval between 23.5 and 56.5 days (after fecundation), that indicates probably an auto-catalytic growth stage during this interval.

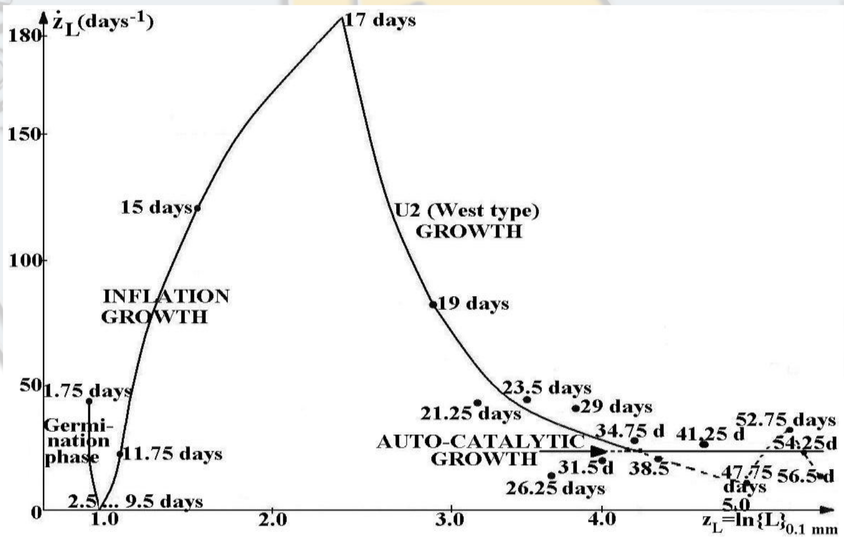

Fig. 9. Basic stages of the human embryo growth.

(ii) Fetal growth

The average values (indicated by references [17a-c]) of the human fetus mass (weight) m (g), were used to calculate the values of the similitude parameter $z_m = \ln\{m\}_g$ and of its growth rate: $\dot{z}_m = \dfrac{m(t+\Delta t) - m(t-\Delta t)}{\tilde{m} \cdot 2\Delta t}$. The obtained values were synthesized by figure 10.

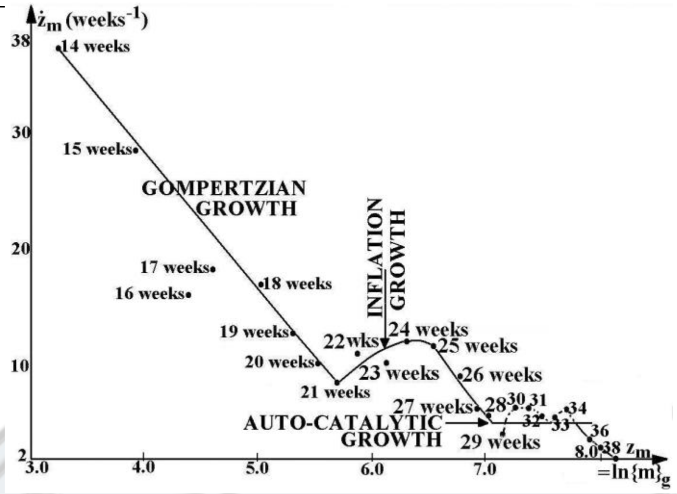

Fig. 10. Basic stages of the human fetus growth

(iii) Baby boy growth

The average values (indicated by reference [19]) of the baby boy length L (cm) and mass m (kg), were used to calculate the values of the similitude parameters $z_L = \ln\{L\}_{0.1\,mm}$ and $z_m = \ln\{m\}_g$, and of the similitude parameters growth rates: $\dot{z}_L = \dfrac{\Delta L}{L_m \Delta t}$ and $\dot{z}_m = \dfrac{\Delta m}{\tilde{m} \cdot \Delta t}$. The obtained results were synthesized by figure 11.

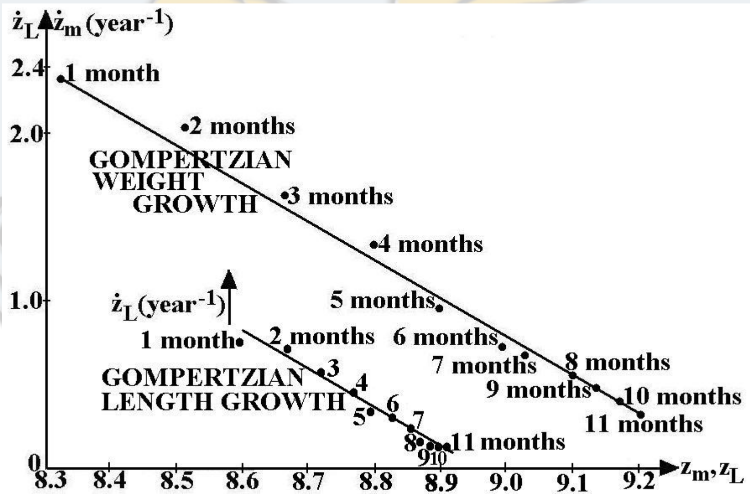

Fig. 11. Basic stages of the baby boy growth

As the length growth or the mass (weight) growth is prevailing in a certain growth stage, the values of the ratio r are less or larger than 3, respectively.

(iv) Boy and teenager (lad) growth

The average values (indicated by reference [19]) of the boys and teenagers heigh H (cm) and mass m (kg) were used to calculate the values of the similitude parameters $z_H = \ln\{H\}_{0.1\,mm}$ and $z_m = \ln\{m\}_g$, and of these similitude parameters growth rates: $\dot{z}_H = \frac{\Delta H}{H_m \Delta t}$ and $\dot{z}_m = \frac{\Delta m}{\widetilde{m} \cdot \Delta t}$.

The obtained results were synthesized by figure 12.

(v) Weight vs Height growth rates

For all: a) baby boy, b) boy, and c) teenager growth stages, the ratio $r = \dot{z}_m / \dot{z}_L$ of the weight and length (height) growth rates was calculated.

As this ratio is larger or it is less than 3, during the considered growth interval, the weight or the length (height) growth, resp. is prevailing.

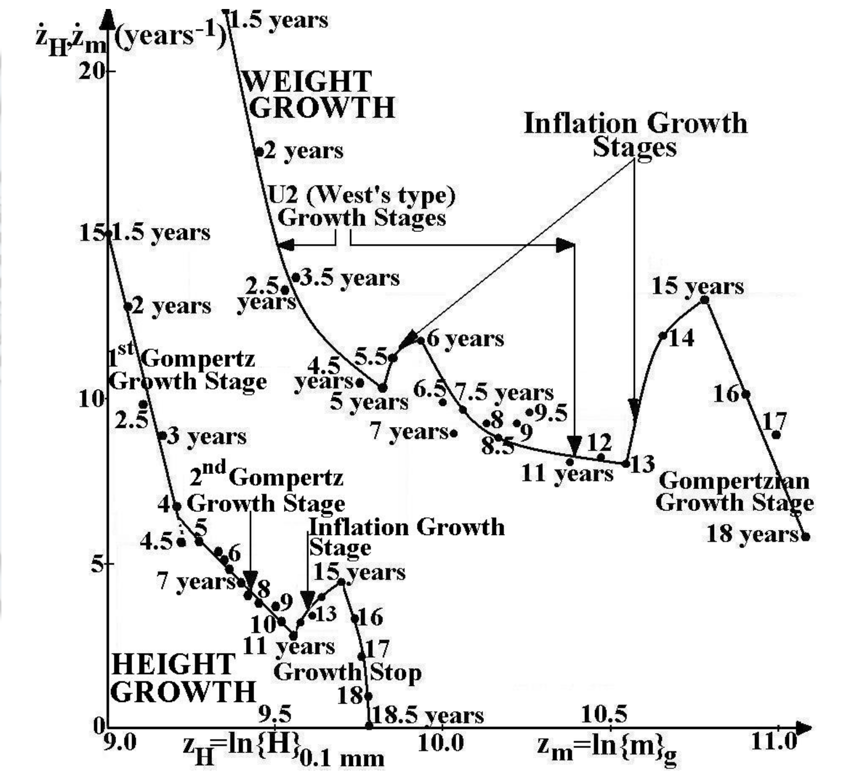

Fig. 12. Basic stages of the boy and teenager growth

Figure 13 presents the time dependence of the calculated ratios of the weight and length growth rates $r = \dot{z}_m / \dot{z}_L$ for the average data [19] corresponding to the baby boys.

Similarly, figure 14 presents the same time dependence for the average data [19] corresponding to the boys and teenagers weight and height growth processes.

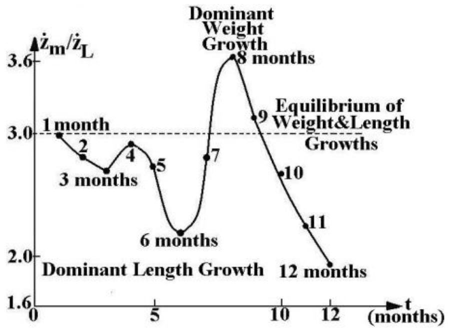

Fig. 13. The time dependence of the ratio of the weight and length growth rates for baby boys

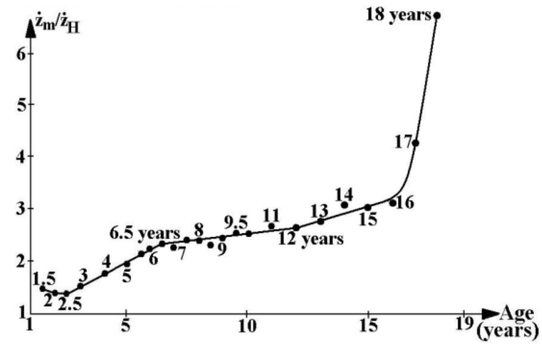

Fig. 14. The time dependence of the ratio of the weight/height growth rates for boys and teenagers

One finds that: *a) excepting the local maximum of the ratio* $r = \dot{z}_m / \dot{z}_L$ located around 8 months (corresponding to a prevalence of the weight growth in this period), there is a rather monotonic decrease of this ratio, indicating the trend of accentuation of the length growth prevalence during this baby stage (see also fig. 13), b) for boys and teenagers, the ratio $r = \dot{z}_m / \dot{z}_H$ has an increasing trend, with some fluctuations leading to the rather weak minims located around the ages of 2.5, 7.0 and 8.5 years, resp. (see fig. 14).

(v) Interpretation possibilities of the results about the human being growth

The different stages of the human being growth represent the consequence of some chemical and biological processes. Their regularities can be explained starting only from the genetic information (see fig. 15) received at the egg fertilization. Of course, the understanding of the connection between the growth features and the genetic information has to be studied thoroughly.

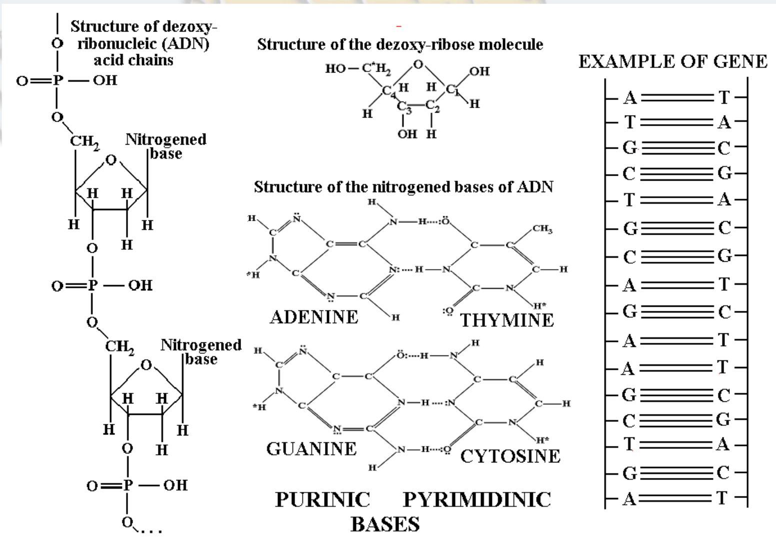

Fig. 15. Chemical structure of the genetic information

Conclusions

The analysis of the obtained results (starting from some existing data [17]-[19] concerning the human body growth) points out that:

a) the use of the similitude "phases space" ($z, \dot{z}$) allows the identification of the main types of growth stages, namely: (i) the auto-catalytic growth processes (the U_0 universality class), (ii) Gompertz's growth (U_1 universality class), (iii) the U_2 (West's type) growth, and of the: (iv) inflation growth processes;

b) despite of their presence in some phases of the human being growth, it seems that the auto-catalytic growth stages are somewhat specific to the adaptation (accommodation) processes [25];

c) the most important stages that initiate the growth processes (the inflation ones) seem to fulfill a power law of the type: $\dot{z} = C \cdot (z - z_i)^n$, where n is a positive irrational number less than 1;

d) because: (i) the same finding seems to be valid also for the Universe inflation stage, corresponding to the present theoretical models of the Universe evolution [20], [21], (ii) the Universe evolution involves also at least a second inflation stage (after its drastic deceleration after the inflation stage, the Universe is again accelerating now [28]), it seems that there are some similarities between the living beings growth and the Universe evolution [27].

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