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# THE STUDY OF THE ELECTROMAGNETIC FIELD FROM THE SHIELDED MICROSTRIP LINE USING THE ELECTRODYNAMIC METHOD

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**Abstract.** *The present article is the first from a series of three papers which will be published in this journal, and which provides a brief presentation of the book, "Microwave - numerical solutions", written by the same author. The main objective of the present article is to describe the development of rigorous mathematical models for the optimal design of microwave circuits which use microstrip lines. The models will use real input data regarding the propagation characteristics of the electromagnetic field and the configuration of all existing wave modes in the line and will provide useful results for customer specific microwave applications.*

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**Keywords:** shielded microstrip line, transverse electromagnetic mode, eigen functions, partial domains.

## I. Introduction

The book "Microwave-Numerical solutions" written by the same author aims to make an essential contribution to the elucidation of many of the challenges of the electromagnetic field in the microstrip line. In addition, the book highlights a suite of interactive program packages designed to study the dynamic behaviours of the electromagnetic field using S-parameters in shielded microstrip line and related microwave circuits.

The present article, the first in a series of three papers which will be published in the present and the next volumes of the "Annals of Academy of Romanian Scientists" is briefly aiming the mathematical description of the electrodynamic method applied to the shielded microstrip line.

In a series of specialized works, [1]÷[7], the analysis and calculation of the parameters of the microstrip lines is carried out under the assumption of the quasi-static approximation, which assumes that the fundamental mode of the propagation wave can be approximated with the transverse electromagnetic mode (TEM ). Such approach allows to obtain satisfactory results only for the values of the longest wavelengths in the microwave frequency range, when the wavelength considerably exceeds the transverse dimensions of the line. Recent achievements in the field of microwaves require, however, the operation of microstrip lines at much higher frequencies, reaching hundreds of GHz, as well as the use of substrates with high relative permittivity. As the working frequency increases, the

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quasi-static analysis of the microstrip line produces increasing errors. This phenomenon is the consequence of the dispersive character of the microstrip line (parameters vary according to frequency) and the existence of higher order wave modes in the line. Since the microstrip line is a non-homogeneous structure, containing air and dielectric domains with different properties, the propagation mode is a hybrid one and cannot be associated with the TEM propagation mode.
The study of the behaviours of the electromagnetic field in the shielded microstrip line assumes the satisfaction of the following objectives, presented in detail in the content of the book:

1) to deal with the real nature of the hybrid propagation modes and determine the higher order hybrid propagation modes and allow to obtain information about the dispersion characteristics of the line parameters;

2) to consider the microstrip line placed inside a metal box, and the conditions generated by electrical shielding effects;

3) to consider, for practical reasons, the fact that the dimensions of the shielding box must be much larger, compared to the thickness of the dielectric domain in the substrate and the width of the metal strip placed between the air and dielectric domains;

4) to use a sufficiently general method to allow obtaining some general solutions, which can be extended to microstrip structures with physical-geometric non-homogeneities of the conductors of more complex lines;

5) to use the right approximations, so that the accuracy of the calculations is limited only by the computing power and related software.

The titles of the next two papers to be published in the coming issues of the Annals, and which will briefly cover the major contributions described in the book are as follows:

- *The study of the electromagnetic field from the shielded microstrip line using the finite difference method*;
- *Matlab software package for calculating parameters of electromagnetic field and microwave circuits*.

**II. Mathematical background of the model**

The analysed method allows verifying the equations of electromagnetism in the entire analysed domain and satisfying the conditions imposed on the electromagnetic field at the separation surface between the air and dielectric domains in the composition of the screened microstrip line and in the immediate vicinity of the edge of the conductor located between them.

## The Helmholtz equations

The formulation of the problem aims to satisfy all the objectives presented in the introductory chapter and the mathematical formalization of the adopted physical model.

The starting point is represented by the scalar equations for the axial components of the electric and magnetic field known as the Helmholtz equations (figure 1):

$$\Delta_T E_z + \mathrm{k}_\delta^2 E_z = 0, \tag{1a}$$

$$\Delta_T H_z + \mathrm{k}_\delta^2 H_z = 0, \tag{1b}$$

and the solutions are determined in the form of series formed by eigen functions, which satisfy, by members, equations (1a) and (1b), respectively:

$$E_{z\delta}(x,y) = \sum_m A_{\delta m}\, Xe_m(x)\ Ye_{\delta m}(y), \tag{2a}$$

$$H_{z\delta}(x,y) = \sum_m B_{\delta m}\, Xh_m(x)\ Yh_{\delta m}(y), \tag{2b}$$

where:

- $A_{\delta m}$ and $B_{\delta m}$ are unknown coefficients;
- $Xe_m(x) = cos\, k_{xm}x$ and $Xh_m(x) = sin\, k_{xm}x$ form a system of eigen (orthogonal) functions on the interval $\left[0,\ \frac{a}{2}\right]$;
- $Ye_{\delta m}(y) = sin[k_{y\delta m}(y - b_\delta)]$ and $Yh_{\delta m}(y) = cos[k_{y\delta m}(y - b_\delta)]$ also form a system of eigen (orthogonal) functions on the interval $\left[0,\ \frac{a}{2}\right]$;
- $k_{xm} = \frac{m\pi}{a}$, $k_{y\delta m}^2 = k_\delta^2 - k_{xm}^2$; the index δ differentiates the two domains in figure 1.

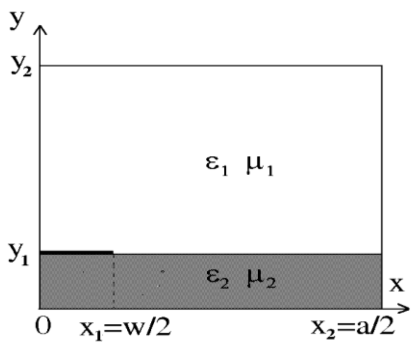


Figure 1. The elementary cell of shielded microstrip line

The transverse components of the electric field and the magnetic field are determined with the relations:

$$E_{x\delta}=-\frac{i}{k_\delta^2}\left(\beta\frac{\partial E_{z\delta}}{\partial x}+\omega\mu_0\mu_{r\delta}\frac{\partial H_{z\delta}}{\partial y}\right) \tag{3a}$$

$$E_{y\delta}=-\frac{i}{k_\delta^2}\left(\beta\frac{\partial E_{z\delta}}{\partial y}-\omega\mu_0\mu_{r\delta}\frac{\partial H_{z\delta}}{\partial x}\right) \tag{3b}$$

$$H_{x\delta}=-\frac{i}{k_\delta^2}\left(\beta\frac{\partial H_{z\delta}}{\partial x}-\omega\varepsilon_0\varepsilon_{r\delta}\frac{\partial E_{z\delta}}{\partial y}\right) \tag{3c}$$

$$H_{y\delta}=-\frac{i}{k_\delta^2}\left(\beta\frac{\partial H_{z\delta}}{\partial y}+\omega\varepsilon_0\varepsilon_{r\delta}\frac{\partial E_{z\delta}}{\partial x}\right) \tag{3d}$$

where $\varepsilon_0\varepsilon_{r\delta}$ and $\mu_0\mu_{r\delta}$ are the dielectric domain's permittivity, respectively magnetic permeability (vacuum and relative ones); in propagation conditions, in the microstrip line without losses, the propagation constant is purely imaginary, respectively: $\gamma\cong i\,\beta$.

**The method of partial domains**

The method of partial domains has gained wider use in solving the most different problems of electrodynamics.

The practical value of the calculation algorithm is determined by its characteristics: the speed of convergence of the chosen solution, the achieved precision, and its stability, which depend on the type of eigen functions used to approximate the solutions of the electromagnetism equations.

To elaborate the selection criteria of orthogonal function systems, a specific situation is analysed.

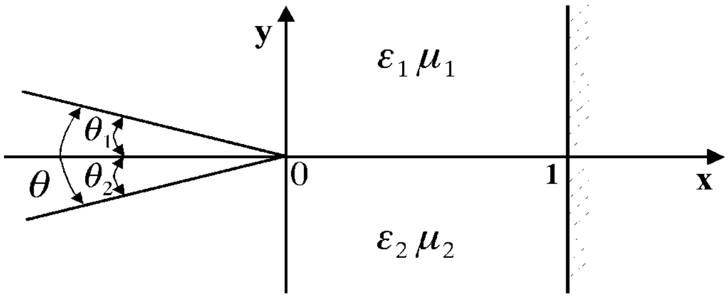


*Figure 2. Model adapted to the possibility of using orthogonal function systems at the separation surface between domains 1 and 2.*

The separation boundary between domains 1 and 2 (figure 2) coincides with the x axis, so that, near the edge, where the electromagnetic field has a particular behaviour, the origin of the axes (x=0, y=0) was placed, and in the x plane =1 there is an electric wall.

The tangential components of the electric and magnetic fields at the separation boundary between domains are used, respectively $E_{z0}$, $H_{z0}$, $E_{x0}$ and $H_{x0}$, expressed in the form of rapidly converging series using orthogonal systems of polynomials and Chebyshev functions, which were defined by means of the relations:

$$E_{z1} = E_{z2} = \begin{cases} 0 & \text{for} \quad 0 \le x \le \frac{w}{2} \\ E_{z0} & \text{for} \quad \frac{w}{2} \le x \le \frac{a}{2} \end{cases} \tag{4}$$

$$H_{z1} = H_{z2} = H_{z0} \text{ for } \frac{w}{2} \le x \le \frac{a}{2} \tag{5}$$

$$E_{x1} = E_{x2} = \begin{cases} 0 & \text{for} \quad 0 \le x \le \frac{w}{2} \\ E_{x0} & \text{for} \quad \frac{w}{2} \le x \le \frac{a}{2} \end{cases} \tag{6}$$

$$H_{x1} = H_{x2} = H_{x0} \quad \text{for} \quad \frac{w}{2} \le x \le \frac{a}{2}. \tag{7}$$

The tangential components of the electric and magnetic fields at the separation boundary between the domains are developed in rapidly converging Fourier series, in relation to the system of orthogonal functions given by polynomials and Chebyshev functions:

$$E_{x0}(x) = \sum_{n=0}^{\infty} C_n \varphi_{en}(x), \tag{8}$$

$$E_{z0}(x) = \sum_{n=0}^{\infty} D_n \psi_{en}(x), \tag{9}$$

$$H_{x0}(x) = \sum_{k=0}^{\infty} F_k \varphi_{hk}(x), \tag{10}$$

$$H_{z0}(x) = \sum_{k=0}^{\infty} G_k \psi_{hk}(x), \tag{11}$$

where $C_n$, $D_n$, $F_k$ and $G_k$ were determined in the contents of the book following an extensive calculation process.

**Electrodynamic expressions of the components of the electromagnetic field in the shielded microstrip line**

The electrodynamic expressions of the longitudinal and transverse components of the electric and magnetic fields in the shielded microstrip line, determined with the help of relations (2a), (2b), (3a)÷(3d) and (8)÷(11) become:

$$E_{z\delta}(\mathrm{x,y}) = \mathrm{A} \sum_m \mathrm{Xe}_m(x) \frac{\mathrm{Ye}_{\delta m}(y)}{\mathrm{Ye}'_{\delta m}(y_1)} \sum_m \alpha_{\mathrm{mn}} d_n, \tag{12}$$

$$H_{z\delta}(\mathrm{x,y}) = \frac{\mathrm{A}k_\delta^2}{k_0 \rho_0 \mu_{r\delta}} \sum_m \mathrm{Xh}_m(x) \frac{\mathrm{Yh}_{\delta m}(y)}{\mathrm{Yh}'_{\delta m}(y_1)} \zeta_{mn\delta}, \tag{13}$$

where:

$$\zeta_{mn\delta} = \sum_{n=1}^{N} \xi_{mn} c_n - \frac{\beta e_m}{k_\delta^2} \sum_{n=1}^{N} \alpha_{mn} d_n,$$

$$Ac_n = C_n, \; Ad_n = D_n,$$

$$\rho_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \;\Omega.$$

and represents the wave impedance, and the relations (3a)÷(3d) become:

$$E_{y\delta}(x,y) = -i\; A \sum_m \mathrm{Xe}_m(x) \left[ \frac{\beta}{k_\delta^2} \frac{\mathrm{Ye}'_{\delta m}(y)}{\mathrm{Ye}'_{\delta m}(y_1)} \sum_{n=1}^{N} \alpha_{\mathrm{mn}} d_n - \right.$$

$$\left. - h_m(x) \frac{\mathrm{Yh}_{\delta m}(y)}{\mathrm{Yh}'_{\delta m}(y_1)} \zeta_{mn\delta} \right] \tag{14}$$

$$E_{x\delta}(x,y) = -iA \sum_m Xh_m(x) \left[ \frac{\beta e_m}{k_\delta^2} \frac{\mathrm{Ye}_{\delta m}(y)}{\mathrm{Ye}'_{\delta m}(y_1)} \sum_{\mathrm{n}=1}^{N} \alpha_{\mathrm{mn}} d_n + \right.$$

$$\left. + \frac{\mathrm{Yh}'_{\delta m}(y)}{\mathrm{Yh}'_{\delta m}(y_1)} \zeta_{mn\delta} \right], \tag{15}$$

$$H_{x\delta}(x,y) = -\frac{iA}{k_0\rho_0\mu_{r\delta}}\sum_m Xe_m(x)\left[\beta h_m \frac{Yh_{\delta m}(y)}{Yh'_{\delta m}(y_1)}\zeta_{mn\delta} - \right.$$

$$\left. - \mu_{r\delta}\varepsilon_{r\delta}\left(\frac{k_0}{k_\delta}\right)^2 \frac{Ye'_{\delta m}(y)}{Ye'_{\delta m}(y_1)}\sum_{n=1}^{N}\alpha_{mn}d_n\right], \tag{16}$$

$$H_{y\delta}(x,y) = -\frac{iA}{k_0\rho_0\mu_{r\delta}}\sum_m Xh_m(x)\left[\beta \frac{Yh'_{\delta m}(y)}{Yh'_{\delta m}(y_1)}\zeta_{mn\delta} + \right.$$

$$\left. + \left(\frac{k_0}{k_\delta}\right)^2 e_m(x)\varepsilon_{r\delta}\mu_{r\delta} \frac{Ye_{\delta m}(y)}{Ye'_{\delta m}(y_1)}\sum_{n=1}^{N}\alpha_{mn}d_n\right] \tag{17}$$

The constant A can be determined from the norming condition of the eigen functions. In this sense, the normalization condition of the average power transmitted through the transverse surface of the shielded microstrip line will be used:

$$\left|\sum_{j=1}^{2}\iint_{S_j}\left(E_{xj}H^*_{yj} - E_{yj}H^*_{xj}\right)dxdy\right| = 1$$

The components of the tangential electric and magnetic fields at the separation boundary between domains, respectively $E_{z0}$, $H_{z0}$, $E_{x0}$ and $H_{x0}$, are determined with the help of polynomial systems and Chebyshev functions constituted in rapidly converging series and defined by means of relations (8)÷(11), in which replaced the infinite Fourier series with finite partial sums, respectively:

$$E_{x0}(x) = A\sum_{n=1}^{N}c_n\varphi_{en}(x),$$

$$E_{z0}(x) = A\sum_{n=1}^{N}d_n\psi_{en}(x),$$

$$H_{x0}(x) = \sum_{k=1}^{K}F_k\varphi_{hk}(x),$$

$$H_{z0}(x) = \sum_{k=1}^{K}G_k\psi_{hk}(x),$$

The tangential components of the magnetic field, $H_{z0}$, at the boundary of separation between domains, defined by the Cartesian product $\{y=y_1\}\times\left[\frac{w}{2}, \frac{a}{2}\right]$, expressed in the form of rapidly converging series, were determined in the contents of the book following an extensive calculation process.

## III. Results

It is used a line configuration, frequently used in complex microwave circuits: *w=1 mm, a=3,5 mm,* $y_1$*=0,5 mm,* $y_2$*=2 mm* and $\varepsilon_{r2}$*= 9* (figure 3). The minimum frequency for this line configuration, at which the propagation constant is purely imaginary, is 38.188 GHz; it means the lossless propagation in the line occurs for working frequencies greater than 38.188 GHz.
The configuration of the transverse components of the electromagnetic field, at the working frequency equal to 38.188 GHz is presented in figures 3 and 4.
The book also presents the configuration of the longitudinal components of the electromagnetic field and other frequencies for which lossless propagation occurs.

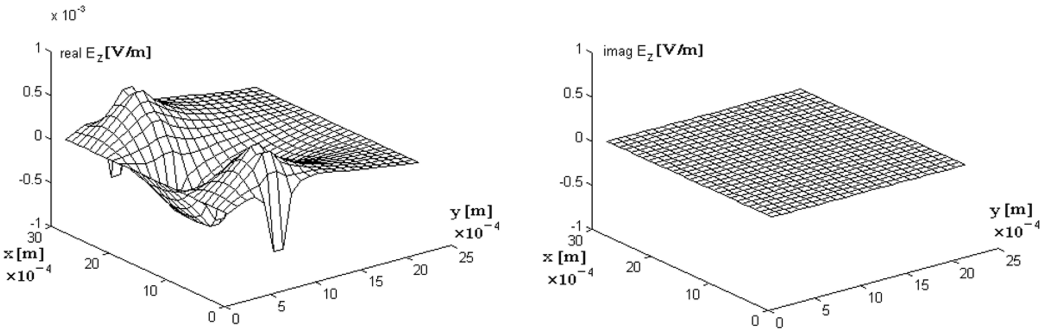


*Figure 3. The variation of the longitudinal component of the electric field in the nodes of the network*

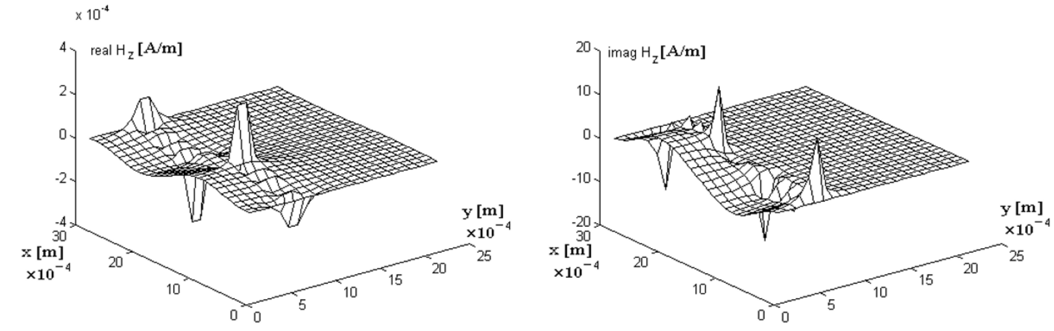


*Figure 4. The variation of the longitudinal component of the magnetic field in the nodes of the network*

## IV. Concluding remarks

The electrodynamic analysis of the electromagnetic field allows the most accurate deciphering of the phenomena existing in the shielded microstrip line under the conditions of satisfying all the objectives presented in the introductory chapter.

Consequently, with the help of the presented method, the configuration of the electromagnetic field, their parameters and dispersion characteristics are determined, under the conditions of checking the Helmholtz equations in the entirety of the analysed domain, but also under the conditions imposed on the electromagnetic field at the separation surface between the two domains and in the vicinity of the edge the conductor between them.
The representations of the electromagnetic field in the shielded microstrip line confirm the expectations regarding the fact that its major variations are in the domain located in the substrate and in the separation zone between the air and dielectric domains. The components $H_x$, $E_y$ and $E_z$ are symmetric, and the components $E_x$, $H_y$ and $H_z$ are antisymmetric in relation to the axis 0y in figure 2, in which the elementary cell of the screened microstrip line was represented.
Another conclusion drawn from the electrodynamic analysis is the fact that the propagation of electromagnetic energy in the microstrip line does not occur without loss at any operating frequency in the entire microwave range.

## R E F E R E N C E S

[1] Schneider M. V., Microstrip Lines for Integrated Circuits, Bell Syst. Tech. J., vol. 48, pages 1421÷1444, May÷June 1969
[2] Yamashita E., Mitra R., Variational Method for the Analysis of Microstrip Lines, IEEE Trans. Microwave Theory Tech., vol. MTT - 16, pages 251÷256, Apr. 1968
[3] Judd S. V., Whiteley I., Clowes R. J., Rickard D. C., An Analytical Method for Calculating Microstrip Transmission Line Parameters, IEEE TMTT, vol. MTT-18, pages 78÷87, Feb. 1970
[4] Stinehelfer H., An Accurate Calculation of Uniform Microstrip Transmission Lines, IEEE TMTT., vol. MTT-16, pages 439÷444, July 1968
[5] Wheeler H. A., Transmission - Line Properties of Parallel Strips Separated by a Dielectric Sheet, IEEE TMTT, vol.MTT-13, pages 172-185, March 1965
[6] Sobol H., Applications of Integrated Circuits Technology to Microwave Frequencies, Proc. IEEE, vol. 59, pages 1200÷1211, Aug. 1971
[7] Sylvester P., TEM Wave Properties of Microstrip Transmission Lines, Proc. IEEE (London), vol. 115, pages 43÷48, 1968
[8] Lojewski G., Microwave devices and circuits, Technical Publishing House, Bucharest, 2005, ISBN 973-31-2263-7
[9] Crisan N., Antennas and circuits for microwaves, Risoprint Publishing House, 2008, ISBN 978-973-751-867-5
[10] Iordăchescu G-A, Microwaves: theory and applications, Publishing House of the University of Pitesti, 2018, ISBN 978-606-560-595-4
[11] Cantaragiu S., Analysis of shielded microstrip lines by finite-difference method, ICECS'99. Proceedings of ICECS'99. 6th IEEE International Conference on Electronics, Circuits and Systems (Cat. No. 99EX357), pages 565-567, 1999/9/5
[12] Cantaragiu Ş., Microwaves circuits, Publishing House All, Bucharest, 2000, pages 1-195, ISBN 973-684-165-0

[13] Coman C.I., Lager I.E., Ligthart L.P., Cantaragiu Ș. and Szilagyi A., The effect of the mutual coupling in smart antenna for electronic warfare applications, NATO IST/SET Symposium, Chester, UK, 7-8 April 2003
[14] Cantaragiu S., Microwaves numerical methods, pages 1-400, Publishing Houses: Academy of Romanian Scientists, ISBN 978-630-6518-03-6 and Tehnică, ISBN 978-973-31-2410-8, MICROUNDE-SOLUȚII NUMERICE MICRO-ONDES – SOLUTIONS NUMERIQUES **** ONLINE VERSION (aos.ro), Bucharest, 2023.