VARIATIONAL PDE-BASED MODELS FOR IMAGE FILTERING AND INPAINTING

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Abstract. This work represents a survey of the main image denoising and inpainting techniques based on variational (enery-based) schemes that lead to nonlinear partial differential equation (PDE) –based filtering models. The variational denoising and restoration approaches are described first. Then, the energy-based structural image reconstruction models are surveyed here. Some of our own variational techniques proposed in these closely related image processing and analysis domains, will be also briefly described in this article.

Keywords: image denoising and restoration, nonlinear PDE-based model, variational technique, structural inpainting, numerical approximation algorithm.

1. Introduction

Image noise removal and inpainting represent important and still-challenging image processing fields that are also closely related. An overview on these domains is provided in this research work.

The image denoising and restoration represents the process of eliminating the electronic noise, which is a random variation of color information or brightness, from the 2D image signal [1]. There exist many types of image noise, the most popular of them being the 2D additive white Gaussian noise (AWGN), which represents a statistical noise having a PDF equal to that of the normal distribution and its major source arises during the acquisition and transmission processes.

Finding an effective noise reduction method still constitues a challenge in the image processing area. Such a filtering scheme has to optimize the trade-off between noise removal, detail preservation and avoiding undesired effects, like blurring or staircasing.

The classic denoising models, such as 2D Gaussian or Averaging filters, generate the blurring that corrupts the edges, corners and other features [1]. Therefore, the partial differential equation (PDE) - based filtering algorithms were introduced to solve properly this issue. The nonlinear PDE-based techniques have proved to be a great solution for this filtering task, providing effective detail-preserving restorations [2, 3]. They perform a directional diffusion that is degenerate along the gradient direction and has a smoothing effect along but not across the image boundaries. Some of these nonlinear diffusion-based models follow variational

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principles and can be represented in energy-based form. They are disscused in the next section, where some of our contributions in this area are also described.
The image *interpolation* (*inpainting*) aims to reconstruct the missing or highly damaged image regions as plausibly as possible, using the information extracted from the surrounding zones. This domain has the origin in the ancient art restoration [4] and has many applications, like digital artwork reconstruction, photo/movie renovation, object removal, image super-resolution/zooming and image compression.
The inpainting methods can be grouped into 3 categories: texture-based, structure-based and combined schemes. The textural interpolation approaches are related to texture synthesis [5, 6], or represent exemplar-based algorithms [7]. The structural inpainting techniques employ information around the missing component in order to estimate the isophotes from coarse to fine, and diffuse that information. They complete the missing image region using variational and PDE-based algorithms [4]. The state of the art variational structural interpolation methods are presented in the third section. We have performed much research in the variational denoising and structure-based inpainting area, developing a lot of novel techniques [8]. Our main contributions to this research field are also described in these sections. This work ends with a conclusions section and the references.

2. Energy-based Noise ReductionTechniques

The PDE-based denoising models following the variational principle are obtained from minimizations of various energy cost functionals. Such a energy functional sums a *regularization* to a *fidelity* component. The generic variational (energy-based) filtering scheme is given by minimization $\min_u \{E(u) = R(u) + F(u)\}$, where $E(u)$ is the energy functional, the regularization term $R(u) = \alpha \int_\Omega \psi(\|\nabla u\|) d\Omega$, ψ is the regularizer function, and the fidelity term $F(u) = \frac{\beta}{2} \int_\Omega (u - u_0)^2 d\Omega$, with $\alpha, \beta > 0$.
Many such variational denoising models have been constructed in the last 30 years. They lead to some nonlinear PDE-based models by applying the Euler-Lagrange equation and next the steepest gradient descent method.
An influential energy-based restoration technique is that introduced in 1992 by Rudin, Osher and Fetami [9]. Their Total Variation (TV) - ROF Denoising model that minimizes the TV norm is very effective at simultaneously preserving the isophotes while filtering away the additive noise in the flat image regions. It has the following form:

$$u^* = \underset{u \in L^2(\Omega)}{\arg\min} \int_{\Omega} \left(\|\nabla u\| + \frac{1}{2\lambda} (u_0 - u)^2 \right) d\Omega \tag{1}$$

where $\lambda > 0$ and u^* represents the restored image. A second-order nonlinear PDE model is then obtained, using the Euler-Lagrange equation and gradient descent [9], as follows:

$$\frac{\partial u}{\partial t} = div\left(\frac{\nabla u}{|\nabla u|}\right) + \frac{1}{\lambda}(u - u_0) \tag{2}$$

This total variation regularization technique removes properly the additive Gaussian noise, but it can be also adapted to other additive noise models. So, the TV Denoising model for the Laplacian noise is the following [10]:

$$u^* = \min_{u \in BV(\Omega)} \int_{\Omega} \left(|\nabla u| + \lambda |u - u_0| \right) d\Omega \tag{3}$$

where $BV(\Omega)$ is the bounded variation image space. The total variation-based filtering model for Poisson noise is the next one [11]:

$$u^* = \underset{u \in BV(\Omega)}{\arg\min} \int_{\Omega} \left(|\nabla u| + \lambda (u - u_0 \log u) \right) d\Omega \tag{4}$$

Many well-known second-order nonlinear anisotropic diffusion-based restoration models can be also described in variational form. Such an influential PDE-based restoration technique is the anisotropic diffusion scheme of Perona and Malik [12], expressed as:

$$\begin{cases} \dfrac{\partial u}{\partial t} = div\left(g\left(\|\nabla u\|^2\right) \cdot \nabla u\right) \\ u(0, x, y) = u_0(x, y) \end{cases} \tag{5}$$

where the *diffusivity* (*edge-stopping*) function $g : [0, \infty] \to [0, \infty]$ is monotonic decreasing, convergent to zero and g (0) = 1. The authors provided 2 such functions: $g(s^2) = e^{-\frac{s^2}{k^2}}$; $g(s^2) = \dfrac{1}{1 + \left(s/k\right)^2}$, $k > 0$. They also proposed a numerical approximation for this PDE model, based on the finite difference method [12]. The P-M technique produces effective additive noise reduction results and preserves the boundaries very well. In the last decades a lot of anisotropic diffusion-based filtering models following the variational principle have been derived from this Perona-Malik approach [2,3].

Although the second-order nonlinear diffusion models, in both PDE and variational form, provide good detail-preserving denoising and deblurring results, they still have their own drawbacks. Such a major shortcoming is the unintended staircase effect that may create flat regions separated by artifact boundaries. So, many variational algorithms improving the total variation regularization model and overcoming this undesired effect have been proposed in the last 25 years.

Thus, the Total Variation Denoising with Split Bregman applies the iterative Split Bregman algorithm for solving the TV minimization problem [13]. An improved variant of the TV-ROF scheme is the TV – l_1 restoration model that is contrast invariant and able to separate the features according to their scales [14]. The Adaptive TV Denoising represents another improved version of the total variation regularizing [15]. Its regularizer component works like a total variation norm at the boundaries while approximating the l_2 - norm in flat and ramp regions in order to overcome the staircase effect. Higher Degree Total Variation (HDTV) model proposed by Hu and Jacob [16], constitutes another variational denoising solution. It minimizes properly the staircasing and ringing artifacts that are quite common to the TV-based schemes, applying new isotropic and anisotropic high-degree TV regularization penalties. The Generalized Total Variation (GTV) regularization model for image denoising, which is a generalized p^{th} power total variation, represents another improved version of TV-ROF Denoising [17].

Unlike the second-order PDE models, the nonlinear fourth-order PDE-based schemes produce piecewise planar images that look more natural and also avoid the staircasing. A very influential nonlinear fourth-order PDE denoising model is the isotropic diffusion-based algorithm of Y. -L. You and M. Kaveh [18]. Their $L2$ - curvature gradient flow method is obtained from a variational scheme minimizing the next energy cost functional:

$$E(u) = \int_{\Omega} f\left(\left|\nabla^2 u\right|\right) dx dy \tag{6}$$

where $\Omega \subseteq R^2$ and f represents an increasing function. The following fourth-order isotropic diffusion model is achieved after applying Euler-Lagrange equation and the gradient descent approach:

$$\begin{cases} \dfrac{\partial u}{\partial t} = -\nabla^2 \left[g\left(\left|\nabla^2 u\right|\right) \nabla^2 u \right] \\ u(0, x, y) = u_0(x, y) \end{cases} \tag{7}$$

there the diffusivity function is $g(s) = \dfrac{1}{1 + \left(\dfrac{s}{k}\right)^2}$ [18]. The You-Kaveh restoration method provides effective Gaussian noise filtering and avoids successfully the image staircasing but, unfortunately, it may also generate the multiplicative *speckle* noise. A despeckling algorithm is provided in [18] to deal with this issue.

Another important nonlinear fourth-order PDE-based denoising technique is the LLT model of M. Lasaker, A. Lundervold and X. C. Tai [19]. It is obtained from the variational scheme:

$$u = \arg\min_u \int_{\Omega} (\left|D^2 u\right| + \frac{\lambda}{2}(u - u_0)^2) d\Omega \tag{8}$$

where $\lambda \geq 0$ and $\left|D^2 u\right| = \begin{cases} \left|u_{xx}\right| + \left|u_{yy}\right|, or \\ \left(u_{xx}^2 + u_{xy}^2 + u_{yx}^2 + u_{yy}^2\right)^{1/2} \end{cases}$. The LLT restoration scheme outperforms the You-Kaveh scheme for an appropriate choice of the λ parameter. It produces a very good feature-preserving noise reduction and overcomes efficiently the staircasing effect [19].

We also proposed many effective variational image filtering approaches that remove the blurring, preserve the image features and avoid the staircasing. They are disseminated in some of our research papers [8, 20]. Let us present here the variational denoising model in [20] given by:

$$u^* = \arg\min_u J(u),\ J(u) = \int_\Omega \left(\varphi_1\left(\|\nabla u\|\right) + \varphi_2\left(\nabla^2 u\right) + \frac{\lambda}{2}\left(u - u_0\right)^2 \right) d\Omega \quad (9)$$

where φ_1 and φ_2 represent 2 properly chosen regularizers. The next well-posed fourth-order PDE-based model is obtained from it:

$$\begin{cases} \frac{\partial u}{\partial t} = div\left(\psi_1^u\left(\|\nabla u\|\right)\nabla u\right) - \nabla^2\left(\psi_2^u\left(\left|\nabla^2 u\right|\right)\nabla^2 u\right) - \lambda\left(u - u_0\right) \\ u(0, x, y) = u_0 \end{cases} \quad (10)$$

where $\psi_1^u, \psi_2^u : [0, \infty) \to (0, \infty)$, $\psi_1^u(s) = \frac{1}{s}\frac{\partial \varphi_1(s)}{\partial s}$, $\psi_2^u(s) = \frac{1}{s}\frac{\partial \varphi_2(s)}{\partial s}$. A finite difference-based numerical approximation scheme is constructed for this anisotropic diffusion model. From the performed method comparison and found that our nonlinear diffusion method outperforms both the conventional and PDE-based denoising schemes. See some denoising method comparison results in Table 1 and Figure 1.

Table 1. The average PSNR values for several variational and classic denoising techniques

Method	This model	Perona-Malik 1	Perona-Malik 2	TV	You-Kaveh	Gaussian	Median
PSNR	28.34(dB)	26.23(dB)	24.91(dB)	21.88(dB)	27.19(dB)	21.37(dB)	24.53(dB)

3. Variational Structure-based Inpainting Approaches

The variational structure-based inpainting techniques reconstruct the images affected by missing or highly deteriorated regions by solving the next generic energy functional minimization:

$$\min_u \left(E(u) = R(u) + \frac{1}{2}\int_\Omega \lambda_D \left(u - u_0\right)^2 d\Omega \right) \quad (11)$$

where the image domain $\Omega \subseteq R^2$, D is the inpainting domain and the *inpainting mask* is provided by $\lambda_D = \lambda \cdot 1_{\Omega \setminus D}, \lambda > 0$.

Figure 1. Barbara image filtered by variational and non-PDE models

While the minimization of the *regularizer* R (u) containing some apriori information from the evolving image, is responsible for image filling task, *fidelity term* $\frac{1}{2}\int_{\Omega}\lambda_D(u-u_0)^2\,d\Omega$ forces the minimizer to remain close enough to u_0 outside the inpainting domain [4, 21]. So, a generic variational image interpolation model is quite close to the generic variational denoising scheme described in the previous section. Any variational restoration model can be adapted for the reconstruction task by applying an inpainting mask.

Various interpolation models can be obtained by considering various regularizer forms. The Harmonic Inpainting is based on the regularizer $R(u)=\int_{\Omega}\|\nabla u\|^2\,dxdy$ [4]. It represents a quite simple energy-based inpainting model that does not satisfy the *connectivity principle*, since it cannot interpolate succesfully along the image gaps, and also provides too smooth results. Other early variational image completion methods were using the Mumford-Shah image segmentation. Such an interpolation scheme uses the Γ - convergence approximation of the Mumford-Shah functional [22] and it is expressed by the next minimization:

$$\min_u\left(\frac{1}{2}\int_{\Omega}\lambda_D(u-u_0)^2\,d\Omega+\frac{\gamma}{2}\int_{\Omega\setminus\Gamma}z_{\in}^2|\nabla u|^2\,d\Omega+\alpha\int_{\Omega}\left(\in|\nabla z_{\in}|^2+\frac{(1-z_{\in})^2}{4\in}\right)d\Omega\right)\quad(12)$$

where $z_{\in}$ represents the edge set signature function. This reconstruction approach

has a low computation complexity and a fast numerical convergence. It preserves the sharpness of the image edges very well.
A very influential variational image reconstruction algorithm is Total Variation (TV) Inpainting of T. Chan and J. Shen [23], which is given by the minimization:

$$\min_{u \in L^2(\Omega)} \left(E[u] = \int_\Omega \alpha \|\nabla u\| dxdy + \frac{\lambda_D}{2} \int_\Omega (u - u_0)^2 dxdy \right) \tag{13}$$

where $\alpha > 0$ and $\lambda_D = \lambda \cdot (1 - 1_D)$. TV Inpainting fills the missing zone by minimizing the first-order total variation while keeping close to the observed image in the known regions. If a low α value is applied, then the filtering is directed mainly to the inpainting area. It leads to the next second-order nonlinear diffusion model, by using Euler-Lagrange equation and the gradient descent:

$$\begin{cases} \frac{\partial u}{\partial t} = \alpha div\left(\frac{\nabla u}{|\nabla u|}\right) - \lambda_D (u - u_0) \\ u(0, x, y) = u_0 \end{cases} \tag{14}$$

The model (14) inpaints successfully and achieves a good connectivity, but not for large image gaps. Obviously, TV Inpainting is closely related to TV Denoising, being obtained from it by introducing the interpolation mask. Total variation based inpainting models of higher orders have been also proposed in the last years. So, the TV^2 Inpainting model is achieved using the second-order total variation as regularizer component: $R(u) = \int_\Omega |\nabla^2 u| dxdy$ [4]. It outperforms TV Inpainting and produces more natural reconstructed images. TV^2 Inpainting provides also a better connectivity, being able to interpolate properly along large image gaps. The first and the second-order TV regularizations could be also combined in order to get improved interpolation results. Such a mixed TV + TV^2 Inpainting model inpaints the image using the minimization [24]:

$$u^* = \arg\min_u \left(\frac{\lambda_D}{2} \int_{\Omega \setminus D} (u - u_0)^2 dxdy + \int_\Omega \alpha(x) |\nabla u| dxdy + \int_\Omega \beta(x) |\nabla^2 u| dxdy \right) \tag{15}$$

where $\alpha(x)$ and $\beta(x)$ represent 2 spatially varying functions that control the completion process. This combined approach outperforms both TV and TV^2 Inpainting. It fills properly large inpainting areas, overcoming the blocky effect, and is solved numerically via the Split Bregman algorithm [24]. Total Generalized Variation (TGV), which is a generalized version of the total variation model that involves higher-order derivatives of *u*, can be also applied for inpainting [25]. Other improved TV-based inpainting techniques are Blind Inpainting using l_0 and TV Regularization [26] and TV Inpainting with Primal-Dual Active Set [27]. Another well-known higher-order variational image interpolation solution is the Euler's Elastica Inpainting proposed by Chan and Shen [28]. Their reconstruction model uses the next regularizing function:

$$R(u)=\int_D w(u)\left(\alpha+\beta\left(\nabla\cdot\frac{\nabla u}{|\nabla u|}\right)^2\right)|\nabla u|dxdy \tag{16}$$

where the coefficients $\alpha, \beta > 0$ control the behavior of this technique and *w* (*u*) is a weighting function depending on the image's histogram. Euler's Elastica Inpainting executes an efficient reconstruction, being able to inpaint large image zones and works for noisy images too [28]. It achieves a much better connectivity than TV Inpainting and other TV-based methods.

Several inpainting examples illustrating the connectivity power of various variational schemes are displayed in Figure 2.

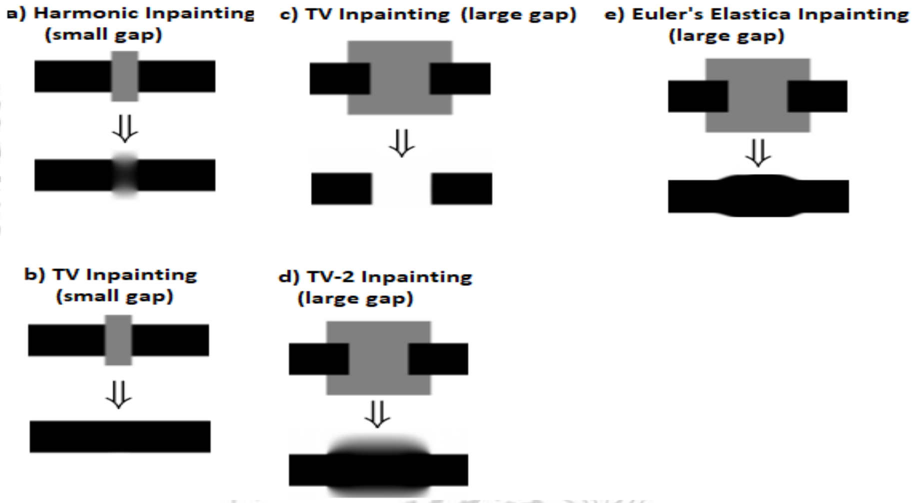

Figure 2. Comparing the connectivity of several variational completion schemes

We proposed many variational structural inpainting techniques in our past research papers [8, 29]. The variational approach in [29] recoveres the affected image applying the minimization $u_{opt} = \arg\min_u F(u)$, where

$$F(u)=\int_\Omega\left(\alpha\varphi_u(\|\nabla u\|)+\frac{\beta}{2}(1-1_\Gamma)(u-u_0)^2\right)d\Omega \tag{17}$$

where the inpainting domain $\Gamma \subset \Omega$ and the regularizer function has the form $\varphi_u(s)=\int_0^s \tau\xi\left(\frac{\eta(u)}{\gamma\log_{10}(s+\eta(u))^2+\delta}\right)^{\frac{1}{3}}d\tau$, with $\eta(u(x,y,t))=\zeta median(\|\nabla u\|)+\nu t$, where $\alpha,\beta\in(0,1)$, $\xi,\gamma,\zeta,\nu\in(0,1]$ and $\delta\in[1,5)$. It leads to the following well-posed nonlinear anisotropic diffusion-based model:

$$\begin{cases} \dfrac{\partial u}{\partial t} = 2\alpha div\left(\psi_u\left(\|\nabla u\|\right)\nabla u\right) - \beta(1-1_\Gamma)\cdot(u-u_0) \\ u(0,x,y) = u_0 \\ u(t,x,y) = 0, \text{on } \partial\Omega \setminus \Gamma \end{cases} \tag{18}$$

where $\psi_u(s) = \xi\left(\dfrac{\eta(u)}{\gamma\log_{10}(s+\eta(u))^2+\delta}\right)^{\frac{1}{3}}$. Its weak solution representing the interpolation output is determined numerically using a consistent fast-converging finite difference method based approximation scheme [30]. This method produces a very good image inpainting in both normal and noisy conditions, preserving the boundaries and other features, and avoiding the unintended effects. The method comparison results are described in Table 2 and Figure 3.

Table 2. The PSNR values obtained by some state of the art inpainting methods

Method	Our model	Navier-Stokes Inpainting [31]	Kriging Interpolation [32]	TV Inpainting	TV Inpainting with PDAS
PSNR	33.14 (dB)	26.42 (dB)	30.38 (dB)	32.85 (dB)	33.96 (dB)

4. Conclusions

An overview of the energy-based image denoising and structural inpainting, which represent two closely related research fields, have been presented in this paper. The two image processing domains are based on generic energy functional cost minimization procedures that are closely related, since the variational restoration models can be adapted for interpolation by introducing an inpainting mask. The described variational denoising and inpainting models lead to some second and fourth-order nonlinear diffusion model that can be solved numerically by applying some iterative numerical approximation algorithms. Many restoration and inpainting techniques surveyed here are using total variation regularizations of first and higher orders. These schemes provide effective feature-preserving noise removal and inpainting results and overcome the undesirable image effects.

Some of our own contributions in these areas have been also described. Our variational techniques achieve satisfactory denoising and reconstruction results, overcome properly the unintended effects and outperform many well-known PDE and non-PDE restoration and interpolation schemes. They could be further applied in other image analysis fields approached by us, such as object detection and recognition [33-35]. Unfortunately, our variational models perform structure-based interpolation only, not working properly for textures. So, the textural inpainting will represent the focus of our future research.

R E F E R E N C E S

[1] Gonzalez, R., Woods, R., Digital Image Processing, Prentice Hall, 2nd ed., 2001

[2] Weickert, J., Anisotropic Diffusion in Image Processing. European Consortium for Mathematics in Industry, B. G. Teubner, Stuttgart, Germany, 1998

[3] Aubert, G., Kornprobst, P., Mathematical problems in image processing: partial differential equations and the calculus of variations, 147, Springer Science & Business Media, 2006

[4] Schonlieb, C. B., Partial Differential Equation Methods for Image Inpainting, 29, Cambridge University Press, 2015

[5] Igehy, H., Pereira, L., Image replacement through texture synthesis, Proceedings of the International Conference on Image Processing, 3, 186–189, 1997

[6] Efros, A., Leung T. K., Texture synthesis by non-parametric sampling, Proceedings of the International Conference on Computer Vision, 2, 1033-1038, 1999

[7] Criminisi, A., Perez, P., Toyama, K. Region filling and object removal by exemplar-based image inpainting, IEEE Transactions on Image Processing, 13 (9), 1200–1212, 2004

[8] Barbu, T., Novel Diffusion-Based Models for Image Restoration and Interpolation, Book Series: Signals and Communication Technology, Springer International Publishing, 2019

[9] Rudin, L., Osher, S., Fatemi, E., Nonlinear total variation based noise removal algorithms, Physica D: Nonlinear Phenomena, 60 (1), 259-268, 1992

[10] Chan, T. F., Esedoḡlu S., Aspects of total variation regularized L1 function approximation, SIAM Journal on Applied Mathematics, 65(5), 1817–1837, 2005

[11] Le, T., Chartrand, R., Asaki, T. A Variational Approach to Constructing Images Corrupted by Poisson Noise, JMIV, 27 (3), 257–263, 2007

[12] Perona P., Malik, J., Scale-space and edge detection using anisotropic diffusion, Proceedings of IEEE Computer Society Workshop on Computer Vision, 16–22, nov. 1987

[13] Getreuer, P., Rudin–Osher–Fatemi Total Variation Denoising using Split Bregman, Image Processing on Line, 2012

[14] Micchelli, C. A., Shen, L., Xu Y., Zeng, X., Proximity algorithms for the L1/TV image denoising model, Advances in Computational Mathematics, 38 (2), 401–426, 2013

[15] Chen, Q, Montesinos, P., Sun, Q, Heng, P., Xia, D., Adaptive total variation denoising based on difference curvature, Image Vision Computing, 28 (3), 298–306, 2010

[16] Hu Y., Jacob, M., Higher degree total variation (HDTV) regularization for image recovery, IEEE Transactions on Image Processing, 21 (5), 2559–2571, 2012

[17] Yan J., Lu, W.-S., Image denoising by generalized total variation regularization and least squares fidelity, Multidimensional Systems and Signal Processing, 26 (1), 243–266, 2015

[18] You Y. L., Kaveh, M. Fourth-order partial differential equations for noise removal, IEEE Transactions on Image Processing, 9, 1723–1730, 2000

[19] Lysaker, M., Lundervold A., Tai X. C., Noise removal using fourth-order partial differential equation with applications to medical magnetic resonance images in space and time, IEEE Transactions on Image Processing, 12, 1579-1590, 2003

[20] Barbu, T., PDE-based Restoration Model using Nonlinear Second and Fourth Order Diffusions, Proceedings of the Romanian Academy, Series A, 16 (2), 138-146, April-June 2015

[21] Song, B., Topics in Variational PDE Image Segmentation, Inpainting and Denoising, University of California, 2003

[22] Ambrosio, L., Tortorelli, V. M. Approximation of functionals depending on jumps by elliptic functionals via Γ - convergence, Comm. Pure Appl. Math., 43, 999–1036, 1990

[23] Chan, T., Shen, J., Morphologically invariant PDE inpaintings, UCLA CAM Rep., 1-15, 2001

[24] Papafitsoros K., Schonlieb, C. B., Sengul B. Combined first and second order total variation inpainting using split Bregman, Image Process. On Line, vol. 2013, 112–136. 2013

[25] Bredies, K., Holler M., A TGV-based framework for variational image decompression, zooming, and reconstruction, Part I: Analytics, SIAM J. Imag. Sci., 8 (4), 2814–2850, 2015

[26] Afonso, M. V., Sanches J. M. R., Blind Inpainting Using l_0 and Total Variation Regularization, IEEE Transactions on Image Processing, 24 (7), 2239–2253, 2015

[27] Neri, M., Zara E. R. Total variation-based image inpainting and denoising using a primal-dual active set method, Philippine Science Letters, 7 (1), 97-103, 2014

[28] Chan, T., Kang, S. H., Shen J., Euler's elastica and curvature based inpaintings, SIAM J. Appl. Math., 2002

[29] Barbu, T., Variational image inpainting technique based on nonlinear second-order diffusions, Computers & Electrical Engineering, 54, 345–353, August 2016

[30] Johnson, P., Finite Difference for PDEs, School of Math., Univ. of Manchester, Sem. I, 2008

[31] Bertalmio, M., Bertozzi A. L., G. Sapiro, Navier-stokes, fluid dynamics, and image and video inpainting, Proc. Conf. Comp. Vision Pattern Rec., 355–362, Hawai, December 2001

[32] Jassim, F. A., Image Inpainting by Kriging Interpolation Technique, World of Computer Science and Information Technology Journal, 3(5), 91-96, 2013.

[33] Barbu, T., An Automatic Face Detection System for RGB Images, International Journal of Computers, Communications & Control, 6 (1), 21-32, 2011.

[34] Barbu, T., Comparing Various Voice Recognition Techniques, 2009 Proceedings of the 5th Conference on Speech Technology and Human-Computer Dialogue, 1-6, 2009.

[35] Barbu, T., An Automatic Unsupervised Pattern Recognition Approach, Proceedings of the Romanian Academy, Series A, 7(1), 73-78, 2006.