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SOME PARTICULARITIES OF SOLITONS PROPAGATION. COMPUTER SIMULATIONS USING MAPLE PROGRAMS

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Rezumat. *Articolul de faţă prezintă unele rezultate numerice ale simulării propagării solitonilor, bazate pe ecuaţiile Korteweg-de Vries (KdV) şi sine-Gordon, utilizând Maple* ***12****, un program puternic, ce permite realizarea calculelor numerice, trasarea şi animarea reprezentărilor grafice 2D şi 3D, modificarea parametrilor, operarea cu expresii analitice. Simulările numerice arată propagarea soluţiilor multisolitonice în fibre optice neliniare, dispersive. Autorul evidenţiază un posibil fenomen legat de recepţionarea datelor şi un artefact numeric. Aceste simulări au fost gândite pentru a reprezenta o bază teoretică necesară atât proiectanţilor din domeniul transmisiei de date cât şi studenţilor, pentru o mai bună înţelegere a fenomenelor.*

Abstract. *This paper presents some interesting numerical simulations of multisoliton propagation, based on the Korteweg-de Vries (KdV) and sine-Gordon equations, using a powerful PC program (Maple****12****) which allows performing numerical calculations, plotting and animate 2D and 3D functions, varying parameters and managing analytical expressions. The numerical simulations show the multisolitonic propagation in a nonlinear dispersive medium. The author shows a possible phenomenon connected with the date reception and a numerical artifact. These simulations are thought to represent a necessary theoretical background both for the designers working in digital data transmission and students, for a better understanding of those phenomena.*

Key words: computer simulations, Maple**12**, solitons, numerical artifacts

1. Introduction

The development of the optical communications began in the early 1960's and continues strongly today; among the inventions that have contributed to the progress of the optical communications the following must be considered as *milestones:* the invention of the LASER (1950's), the development of low loss optical fibers (Corning Optical Fiber, part of Corning's Telecommunications, 1970's), the invention of the optical fiber amplifier (e.g. erbium-doped fiber amplifier, semiconductor optical amplifiers 1980's), the invention of the in-fiber Bragg grating (see [1, 2], 1990's), and also the solitonic transmission of data (1990's).

In the last years, many authors - using different methods and computer programs to simulate various types of solitons - were able to predict some new types of solitons and some interesting features of solitonic propagation [3-9].

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Several new kinds of solitons have been discovered by this means [7-12]; among these researchers there are a few who also use various issues of **Maple** for simulating.

We began our numerical and graphical analyses of the solutions of the KdV equation using **Maple11** in the frame of work [13]. This paper deals with the solitonic solutions of the Korteweg-de Vries equation (KdV), discusses some 3D graphics and 2D animated graphs for the solitonic solution of KdV equation, and reveals two possible phenomena: one phenomenon is numerical; the other is connected with the optical data transmission.

The fundamental books used for this paper are [14-19] and the data were collected from [20-29].

2. Equations for solitons

The model used is based on the well known Korteweg-de Vries equation:

$$\frac{\partial \mathrm{u}(x,t)}{\partial t}+6\,\mathrm{u}(x,t)\frac{\partial \mathrm{u}(x,t)}{\partial x}+\frac{\partial^3 \mathrm{u}(x,t)}{\partial x^3}=0 \qquad (1)$$

and sine-Gordon equation:

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\mathrm{u}(x,t)\right)-\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\mathrm{u}(x,t)\right)-\sin(\mathrm{u}(x,t))=0 \qquad (2)$$

3. Numerical simulations

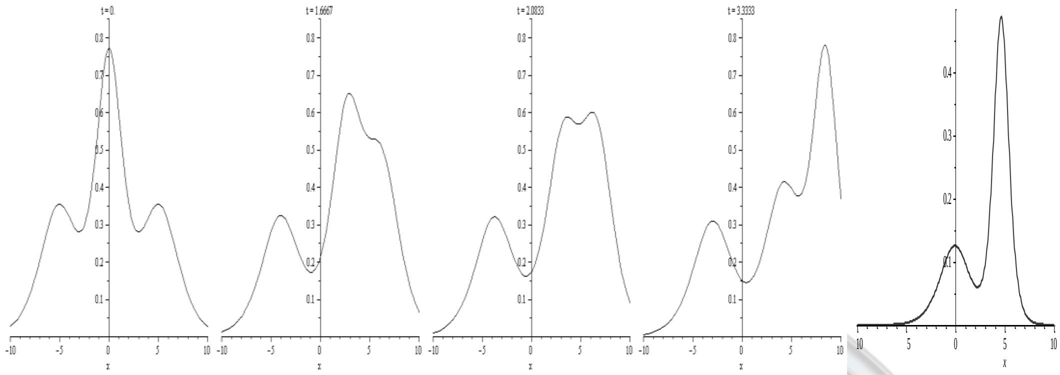
The author has performed various types of solitons propagation numerical simulations; some interesting aspects occurred.

The numerical simulations were performed using the features of Maple12, whose powerful code for equation solving is competitive with the codes written by hand (like e.g. in [3], fig. 2.a, b; the authors deal in this paper with the nonlinear Schrödinger equation solitonic solutions), the author was able to identify some peculiar aspects of some of these plots.

The simulations shown below are 2D for the trisolitonic solution of the KdV eq. (see Figure 1a.) and 3D for the solitonic solutions of sine-Gordon eq. (see Figures 2, 3, and 4).

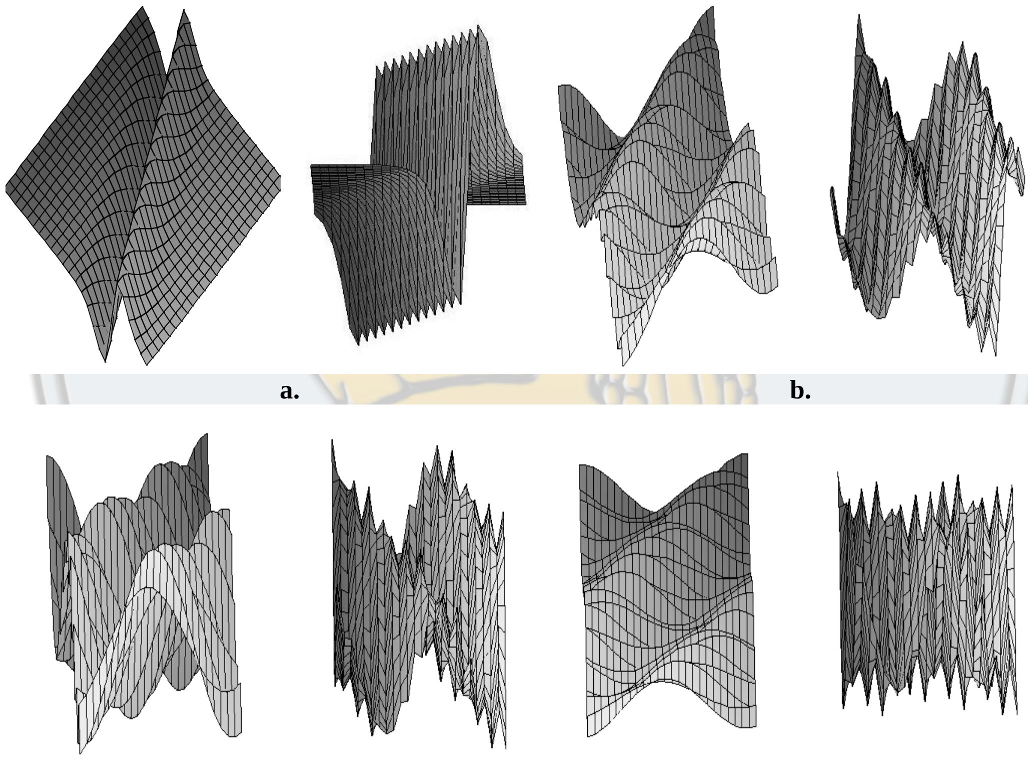
For the 2D simulation of *trisoliton* solution one may observe that this solution takes a form compatible with *two solitons* simulation (see Figure 1.b.).

For the 3D simulation, one may observe that the edge of the envelope seems *smooth* from a certain angle, but appears chirped from another, as a *fractal*; at whatever scale the aspect remains the same.



a. **b.**

Fig. 1. Animated plot for the trisolitonic solution (a) and for the two solitonic solution (b).



a. **b.**

c. **d.**

Fig. 2. 3D implicit plots for the solution of sine-Gordon eq. (a) and KdV equation (b, c, d).

Conclusions

Those computer experiments are very suitable for demonstrating to students in optical engineering and designers of optical circuits the main features of solitons propagation in optical fibers, as it follows.

For designers working in digital data transmission, the simulation of the solitons propagation along optical fibers is useful and may be performed successfully with **Maple12**. The animated plots (also shows some useful features of the solitons collision (for three soliton collision, see fig. 1.a.).

As known, solitons keep invariant shape and size in accordance with the conservation laws, so one can infer that a profound link exists between integrable models and the theory of solitons.

By simulating, we observed that for certain moments the trisolitonic solution is shaped like the two-solitonic solution (see fig. 1.b.); so, while the signals are received the detector will miss some data. In practice, the speed of transmission is chosen so that the gap between solitons arriving is large enough (six time the half width), but this phenomenon will still occur. For determining what the case is, one may calculate (with Maple12) the skewness (which, for symmetrical distributions must be zero).

Also, we observed that the edge of the soliton envelope appears in fact not so smooth; on the contrary it shows a *fractal-type* edge (see fig. 2. a-d); this is a numerical artifact and the simulations may take it into account, to avoid misinterpretations.

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